PART SECOND.

OLNEY'S PRACTICAL ARITHMETIC

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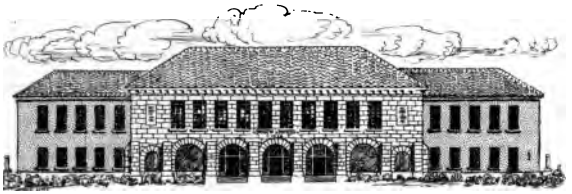

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disciplined who should
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Professor Moses Coit Tyler, though a much younger man than Professor Morley, has been also for many years a practical teacher of English Literature to advanced students in a great university; has had a varied and successful career in general authorship; and especially by his elaborate "History of American Literature," has come to sustain a relation to literary history in this country similar to that held by Professor Morley in England. The combined labors of two such men ought to give us the long-needed Text-Book in English Literature.

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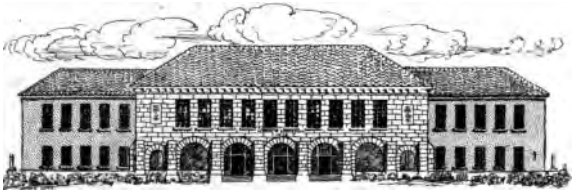

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PREFACE.

MULTIPLICITY of wants and diversity of tastes are characteristics of advanced civilization. The wigwams of a savage tribe are as uniform in their structure as the nests of a species of birds; and in dress the savages are well-nigh as homogeneous as the birds. On the other hand, scarcely two dwellings in an enlightened community are precisely alike; while endless diversity is seen in the costumes of the inhabitants of the same city. So with intellectual demands. The time has gone by when a single Spelling-Book, a single Reader, and a single Arithmetic will be accepted throughout the country. The necessities of schools are varied, and the views and methods of good teachers are not the same. In reference to Arithmetic, for example, we have every variety of view, — from that which would make the mere *modus operandi* the sole aim, to that which confines attention almost exclusively to the study of principles and to the analysis, assuming that the pupil well grounded in these cannot be greatly deficient in practical operations.

It is this diversity of demand that has led to the preparation of the text-book on Arithmetic now offered to the

public. While the Author's "ELEMENTS OF ARITHMETIC" has, in a very flattering way, met the wants of teachers who desire to make the study of principles especially prominent, there are still many teachers whose views of the philosophy of teaching, or whose circumstances, demand a less theoretical and more characteristically practical treatise. Such teachers will find their wants met by this book. All statements of principles, definitions, and rules are reduced to the most brief and simple language consistent with clearness and accuracy; and all demonstrations, illustrations, and methods of solution are made as explicit, direct, and practical as possible.

While the examples are taken, in large part, from the "ELEMENTS," some of the more complicated of the latter have been omitted, and a great number of ordinary practical exercises have been added. *As a book of work, it is believed to be richer than any hitherto offered to the public.*

The *Metric System* is presented somewhat earlier and somewhat fuller than in the "Elements;" and a special chapter on *Mensuration* has been added. In fact, the Author has availed himself of the suggestions of many friends, who have examined the "Elements," and have desired a book containing the same freshness and practical adaptation of problems, and the same thoroughness in the exposition of principles, but have at the same time desired a book which should be somewhat less a development of a method of teaching, and more characteristically a *book of work* for the pupil. With a view to meeting the practical wants of the schoolroom in our great graded schools, this

book, at every step of its progress, has been closely scrutinized, and vigorously criticised by one of our most intelligent, experienced, and successful teachers.

As in the former volume, so in this: the methods known as "Mental" have been carefully incorporated with those known as "Written," in order to present the subject of Arithmetic in one volume as fully and completely as the needs of our public schools require. Thus, for the great body of our schools, but one book in Arithmetic will be required after the primary course.

The Author feels himself specially fortunate in securing publishers who bring out his books in the best style of the printer's art, and on the best material. The good taste of the stereotypers, the elegance of the engraved chapter-headings, the perfection of the press-work, the quality of the paper, the simple elegance and firmness of the binding, are mechanical features of no small moment in a text-book, and which in this cannot fail to challenge admiration.

EDWARD OLNEY.

UNIVERSITY OF MICHIGAN,
ANN ARBOR, July, 1879.

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SECTION I.

DEFINITIONS AND TABLES.

183. An Abstract Number is a *mere* number; that is, a number not applied to any specified things. Thus *ten*, *seven*, 146, $\frac{2}{3}$, $4\frac{3}{4}$, are abstract numbers.

184. A Concrete Number is a number applied to some specified thing; as *ten* men, *seven* trees, 146 feet, $\frac{2}{3}$ of an acre, $4\frac{3}{4}$ pounds, etc.

185. Denominate Numbers means, literally, *Named Numbers;* but the term is applied only to concrete numbers which represent *money*, *weight*, or *measure*. Thus \$5, 10 *lb.*, 3 *gal.*, are denominate numbers; but 5 *men*, 10 *trees*, 3 *stones*, are not so called.

186. In denominate numbers, the different *Orders*, as of money, weight, or measure, are called **Denominations.** Thus *dollars*, *dimes*, and *cents* are denominations of money; and *rods*, *feet*, and *inches* are denominations of measure.

187. A Compound Number consists of several related denominations written together, and to be read as one number. Thus 4 *gal.* 2 *qt.* 1 *pt.* is a compound number; so also is 6 *mi.* 25 *rd.* 10 *ft.*

MEASURES OF VALUE.

188. **Money** is the measure of value used for purposes of buying and selling.

189. **Coins** are pieces of metal bearing the government stamp, and having a value fixed by law. Coin is also called *Specie*. (See Appendix for cuts of coins in most common use.)

190. **Currency**, as a business term, means notes or bills issued by the government or by banks to be used as money.

FEDERAL OR UNITED-STATES MONEY.

191. The **Denominations** of United-States money are eagles, dollars, dimes, cents, and mills; the abbreviations indicating each being, respectively, E., \$, d., ¢ or ct., and m. For ordinary purposes, amounts are mentioned in dollars and cents.

192. **Table.** 10*m.* = 1¢, 10¢ = 1d., 10d. = \$1, \$10 = 1 eagle. Hence we see that United-States money is decimal.

193. The coins of the United States at present struck (1879) are, of *Gold*, \$1, \$2½, \$3, \$5, \$10, and \$20 pieces; of *Silver*, dime, quarter-dollar, half-dollar, and dollar pieces; of *Nickel*, 5-cent and 3-cent pieces; of *Bronze*, a 1-cent piece.

194. The *Trade Dollar*, 20-cent, 5-cent, and 3-cent pieces, of silver, and the copper 2-cent and 1-cent pieces, though found in circulation, are not now coined. The mill was never coined.

195. None of the present United-States coinage is pure metal. The (so-called) *gold* is .9 gold and .1 copper and silver; the *silver* is .9 silver and .1 copper; the *nickel* is ¾ copper and ¼ nickel; the *bronze* is .95 bronze and .05 tin *and zinc.*

196. The **Legal Weights** are, the *gold dollar* 25.8 grains, and the other gold coins in proportion. The legal *silver dollar* weighs 412½ grains, and the *trade dollar* 420 grains. The subsidiary silver coins are on the basis of 385.8 grains to a dollar. The *nickel* 5-cent piece weighs 5 grams (metric weight) or 77.16 grains, and the 3-cent piece 30 grains. The *bronze* cent weighs 48 grains. When gold coin comes into the United-States Treasury reduced in weight more than .005, it is recoined.

MONEY OF FOREIGN COUNTRIES.

197. The Denominations and Values of **Canada Coins** are similar to those of the United States; and Canada currency is reckoned in dollars and cents, though reckoning in English currency is still common.

198. The Denominations of **English Money** are pounds, shillings, pence, and farthings, represented respectively by £, *s.*, *d.*, and *far.*

The *gold coin*, whose value is £1, is called a *sovereign* (*sov.*). Its value in United-States gold is $4.8665. From this value and the following table the pupil will be able to calculate the value of any English coin.

199. Table. 4 *far.* = 1*d.*, 12*d.* = 1*s.*, 20*s.* = £1.

A *guinea* is a gold coin equal to 21*s.* A *crown* is a silver coin equal to 5*s.* A *florin* is a silver 2-shilling piece.

200. The **French Coins** most frequently mentioned in this country are the *franc* (*frank*), the *Napoleon*, and the *centime* (*centeem'*).

The *centime* is $\frac{1}{100}$ of a franc, just as our cent is $\frac{1}{100}$ of a dollar. A *Napoleon* is a 20-franc gold piece. The franc is silver, and the centime bronze. The abbreviation for francs is ^{fr}, and centimes are written as decimals; thus 5^{fr} .17 is 5 francs and 17 centimes. A *franc* = 19.3¢ United-States gold.

201. The standard Denominations of **German Coins** are *the mark* (23.8 *cents*), and *the pfennig* $= \frac{1}{100}$ of a mark.

The Prussian *silver thaler* (74.6¢) and the *silver groschen* (2½¢) are coins frequently referred to in this country, the mark and pfennig having been made the standards in 1872.

Examples.

1. What is the value in our coin of an English shilling? Of a penny? Of a farthing? Of a crown? A guinea?

2. What is £5 12*s.* in dollars and cents?

3. What English coin is very nearly a half-dollar? What is very nearly a half-eagle?

4. What is 10*s.* 6*d.* in our coin? What 5*s.* 10*d.*?

5. What is the value in our coin of 1 centime? Of a Napoleon?

6. How near is a 5-franc piece to $1?

7. The French *gold* coins are 100, 40, 20, 10, and 5 franc pieces. How much is each in our coin?

8. The French *silver* coins are 5, 2, and 1 franc pieces, and 50 and 25 centime (i.e., half and quarter francs) pieces. How much is each in our coin?

9. The *bronze* French coins are 10, 5, 2, and 1 centime pieces. How much is each in our coin?

10. 1000000^{fr} are how many dollars and cents? 25^{fr}.37? 15^{fr}.75? 258^{fr}.60? 100^{Nap}? $550 = how many francs?

11. The German *gold* coins are 5-mark, 10-mark, and 20-mark pieces. What is the value of each in our coin?

12. The German *silver* coins are 20-pfennig, 1-mark, 2-mark, and 5-mark pieces. What is the value of each in our coin?

13. The German *copper* coins are 5-pfennig and 10-pfennig pieces. What is the value of each in our coin? *How many* pfennigs make a cent?

14. $100 gold = how many marks?

15. How many pfennigs make $\frac{1}{4}$ of a dollar?

16. How many marks make $1 gold?

202. Chicago **Quotations of Foreign Exchange** are to-day: "London 4.88¼, Paris 5.15⅛, Hamburg 95¾." This means that you can buy a *Draft* (i.e., an order) on a bank in either of these foreign cities at these rates. The "London 4.88¼" means that you will have to pay $4.88¼ for £1. The "Paris 5.15⅛" means that you will have to pay at the rate of $1 for 5fr.15⅛. The "Hamburg 95¾" means that you will have to pay at the rate of 95¾¢ for 4 marks.

17. According to the above quotations, how much will a draft on Hamburg cost me which will pay for a book which is offered me there at 175 *M.*?

18. At the above rates, what will a draft on London cost which will pay a bill there of £125 12*s.*? On Paris, to pay a bill of 1000fr?

MEASURES OF EXTENSION.

203. **A Point** is a place without size.

Points are designated by letters; as D, C, M, N, etc.

204. **A Line** is the path of a point in motion.

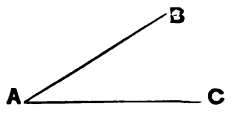
A line is designated by letters placed at its ends; as the line DC, the line MN, etc.

205. **A Straight Line** is the path of a point moving all the time in the same direction. Generally, when we say "Line," we mean a "straight line."

A *Line*, or *Distance*, is *measured* by another line or distance.

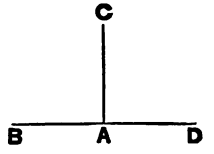
The lines, or distances, commonly used in measuring length, or distance, are an *Inch*, a *Foot*, a *Yard*, a Rod, and a Mile.

206. An Angle is the opening between two lines which meet. The point where the lines meet is called the **Vertex** of the angle.



In common language we call an angle a *Corner*. Thus the corner or opening between the two lines BA and CA is the angle BAC. In designating an angle we use a letter placed at the vertex, or three letters, one on each of the lines, with the letter which stands at the vertex named between them. Thus the angle above may be spoken of as the "angle A," or the "angle BAC."

207. When one straight line, as CA, meets another, as BD, so as to make the angles CAB and CAD equal (i.e., just alike), the angles are called **Right Angles.** A right angle is a *square corner*.



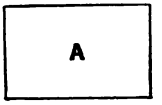
208. An Acute Angle is an angle which is less than a right angle, and an **Obtuse Angle** is one which is greater than a right angle.

For the method of measuring angles, see *Circular Measure*.

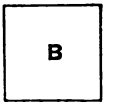
---

209. A space which has length and breadth, but not thickness, is called a **Surface.**

210. A Rectangle is a plane (flat) surface or figure bounded by four straight lines, and having all its angles right angles. A is a rectangle.



211. A Square is a rectangle having all its sides equal each to each. B is a square.



A Surface is measured by another surface, — usually by a square.

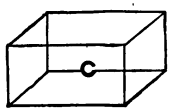
The **Area** of a surface is the number of times it contains the measure.

The surfaces commonly used for measuring surface are the *Square Inch, Square Foot, Square Yard, Square Rod, Square Mile,* and the *Acre.*

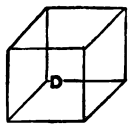
---

212. A **Solid,** or **Body,** is a space having length, breadth, and thickness.

213. A **Right Parallelopiped** is a solid, or space, bounded by six rectangles, and having all its angles right angles. C represents a parallelopiped.



214. A **Cube** is a parallelopiped having all its faces (sides) squares. D represents a cube. A cube, each of whose edges is 1 inch, is a *Cubic Inch;* one whose edges are each 1 foot is a *Cubic Foot;* one whose edges are each 1 yard, a *Cubic Yard,* etc.

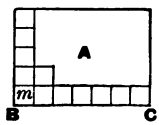


A *Solid is measured* by another solid, — usually by a cube. The **Volume** or **Contents** of a solid is the number of times it contains the measure.

---

215. **Principle.** — *The* **Area** *of a* **Rectangle** *is the product of its length by its width.*

ILLUSTRATION. — If surface A is 7 long and 5 wide, and *m,* the measure, is 1 on a side, we can apply 7 of the measures along one side of the surface, as from B to C, making a row of 7 measures. Now, we can apply the measure so as to make as many such rows as there are units in the width, — in this case 5; and 5 times 7 measures (*m*) make 35 measures. Hence the area of A is 35, *m* being the measure.



1. A square foot is 12 inches on a side. How many square inches in a square foot?

2. A square yard is 3 feet on a side. How many square feet in a square yard?

3. Show that there are $30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ square feet, in a square rod.

4. Show that there are 102400 square rods in a square mile. 160 square rods make an acre. How many acres in a square mile?

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216. Principle. — *The* **Volume** *of a* **Right Parallelopiped** *is the product of its three dimensions; that is, of its length, breadth, and thickness.*

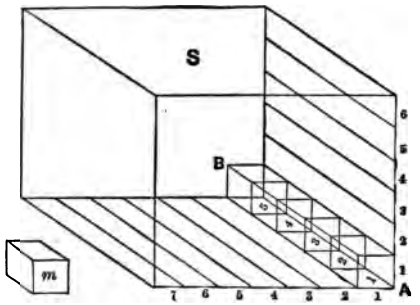

ILLUSTRATION. — If solid **S** is 7 long, 5 wide, and 6 high, and *m*, the measure, is a cube 1 on each edge, we can place 5 of the measures along one side of the bottom (base) of the parallelopiped **S**, as from **A** to **B**, and 7 such rows will cover the base. Thus it takes 7×5 of the measures to cover the base 1 deep. Then, as the parallelopiped is 6 high, it will take 6 such layers, or $7 \times 5 \times 6$, to fill the whole space. Hence the volume of the parallelopiped **S** is $7 \times 5 \times 6$, or 210, *m* being the measure.

1. How many cubic feet in a cubic yard?

2. How many cubic feet in a parallelopiped 8 feet long, 4 feet wide, and 4 feet high? This is called a *cord* of wood.

3. When 4 ft. wood is piled 4 ft. high, 1 ft. in length of pile is called a *cord foot*. How many cubic feet in a cord foot? How many cord feet in a cord?

4. How many cubic inches in a cubic foot?

217. Abbreviations. — *in.* stands for inch or inches, *ft.* for foot or feet, *yd.* for yard or yards, *rd.* for rod or rods,

mi. for mile or miles, *sq.* for square, *cu.* for cubic, *A.* for acre, *cd.* for cord.

218. *Tables of Measures of Extension.*

Long Measure.	Square Measure.	Cubic Measure.
12 in. = 1 ft. ∴	144 sq. in. = 1 sq. ft., and	1728 cu. in. = 1 cu. ft.
3 ft. = 1 yd. ∴	9 sq. ft. = 1 sq. yd., and	27 cu. ft. = 1 cu. yd.
5½ yd. = 1 rd. ∴	30¼ sq. yd. = 1 sq. rd.	16 cu. ft. = 1 cd. ft.
320 rd. = 1 mi.	160 sq. rd. = 1 A.	8 cd. ft. = 1 cd.
	640 A. = 1 sq. mi.	128 cu. ft. = 1 cd.

219. On the common carpenter's square the inch is usually divided into *halves, quarters, eighths,* and *sixteenths,* or into *twelfths.* On other scales it is often divided into *tenths.* A *line* is $\frac{1}{12}$ of an inch.

A *size,* as used by shoemakers, is $\frac{1}{3}$ of an inch, sometimes called a *barleycorn. Children's sizes* run from *size* 1, $4\frac{1}{3}$ in. long, to *size* 13, $8\frac{1}{3}$ in. long. *Youth's, women's,* and *men's sizes* run from *size* 1, $8\frac{1}{3}$ in., to *size* 15, $13\frac{1}{3}$ in.

The ancient *Roman mile* (*mille passum,* 1000 paces) was about 1618 yd., and hence a little shorter than ours. The modern *Roman mile* = .925 *Eng. mi.* The *Irish mile* = 1.273 *Eng. mi.* The *French mile* (*mille marin*) is the same as our marine or geographic mile. The *German short mile* (*meile*) = 3.897 *Eng. mi.;* the *long mile* = 5.753 *Eng. mi.;* the *Prussian mile* = 4.68 *Eng. mi.*

A *geographic, nautical, or marine mile* is 1′ of the equator or of a meridian, and hence is 1.1527 *Eng. mi.* very nearly; a degree being 69.164+ *Eng.* or *statute miles.* A *league* is 3 marine miles. A *furlong* is $\frac{1}{8}$ mi. See table of *Circular Measure.*

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220. **Cloth,** ribbons, laces, etc., are sold by the yard in length, irrespective of the width; the differing widths being considered in fixing the price. For such measurements the yard is divided into halves, quarters, and eighths.

Ex. How many inches in half a yard? How many in a quarter? An eighth? A sixteenth?

221. Sea-depths are measured in fathoms. A **Fathom** is 6 feet.

1. How many feet in a mile? How many fathoms in a mile? In 3 miles?

2. At a certain place the sea was reported as 900 fathoms deep. How much more than a mile deep was it?

3. How many miles in depth is that place in the Atlantic Ocean which is reported as 2640 fathoms deep?

4. In our Western forests many of the trees are 100 feet high. How many fathoms deep would a lake be which would submerge these standing forests?

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222. A Hand is 4 in., used in measuring the height of horses.

1. How many feet high is a horse which measures $15\frac{1}{2}$ hands? 17 hands? 18? 12?

---

223. A Pace is reckoned at 3 ft., although in pacing long distances 5 paces are reckoned a rod.

Ex. How many paces in a mile?

224. For measuring land, surveyors use a *Chain* 4 rods long, and made of 100 links of equal length.

1. How many feet in 4 rods? How many inches in 66 feet? Then how long is 1 link of the surveyor's chain?

2. How many rods in a mile? Then how many chains in a mile?

225. The *Public Lands* of the United States which have been surveyed during the present century have been laid out in **Townships**, which are squares 6 miles on a side. These are divided in what are called **Sections.** A section is a square mile.

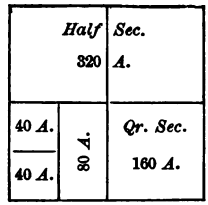
3. How many sections in a township?

4. How many acres in a section? In a half-section? A quarter-section? An eighth of a section?

5. What part of a section is 320 acres? 240? 160? 120? 80? 40?

6. How many 80-acre lots in a section? A half-section? A quarter-section?

7. This figure represents the way in which a section is usually divided. How long and how wide is a quarter-section, or 160 A.? 80 A.? 40 A.?



[*See Appendix* for the method of our Public Land Surveys; also the section on *Mensuration* for applications to the problems of measuring surfaces and solids; as boards, wood, etc.]

MEASURES OF CAPACITY.

226. The Capacity of a vessel is the amount which it contains, — its *contents*.

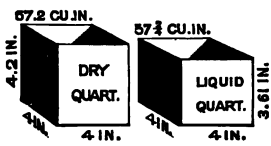
There are two varieties of measures of capacity in common use; viz., *Liquid Measure* and *Dry Measure*.

227. Liquid Measure is used in measuring liquids, or in estimating the capacity of vessels designed to contain liquids, as water, milk, oil, molasses, alcohol, etc. The *Denominations*, or measures, are *gills* (*gi.*), *pints* (*pt.*), *quarts* (*qt.*), *gallons* (*gal.*), and *barrels* (*bbl.*).

228. Dry Measure is used in measuring grain, seeds, fruit, etc., or in estimating the capacity of vessels designed to contain such articles. The denominations are *pints* (*pt.*), *quarts* (*qt.*), *pecks* (*pk.*), and *bushels* (*bu.*).

229. The denominations of like name in these two measures do not represent the same amounts. The *Dry pint* and *quart* are about $\frac{1}{4}$ larger than the *Liquid pint* and *quart*; or, more exactly, the *Dry quart* contains 67.2 cubic inches, and the *Liquid quart* 57.75 cubic inches.

ILLUSTRATION. — These two figures represent boxes, each of which is 4 inches square on the bottom; but the *Dry quart* is 4.2 inches deep, while the *Liquid quart* is but 3.61 inches deep.



1. 4 quarts make a gallon. How many cubic inches make a gallon?

2. 8 quarts make a peck, and 4 pecks make a bushel. How many cubic inches in a bushel?

3. Which is the most, $1\frac{1}{4}$ cu. ft., or 1 bu.?

ANS., The difference is less than $\frac{1}{3}$ pt. ($\frac{2}{7}$).

230. ***Tables of Measures of Capacity.***

LIQUID MEASURE.	DRY MEASURE.
4 gi. = 1 pt.	
2 pt. = 1 qt.	2 pt. = 1 qt.
4 qt. = 1 gal.	8 qt. = 1 pk., or 2 gal.
$31\frac{1}{2}$ gal. = 1 bbl.	4 pk. = 1 bu.

Barrels are made of various sizes, from 30 to 40 or even 56 gallons; but in estimating the capacity of cisterns, vats, etc., $31\frac{1}{2}$ gal. is usually considered a barrel. There is no definite measure in use called a hogshead. Any large cask is freqently so called.

231. Physicians and apothecaries use a kind of liquid measure, of which the denominations are *Minims* (℥), *Fluid Drachms* (f ʒ), *Fluid Ounces* (f ℥), *Pints*, and *Gallons*. The pint and gallon are the same as the common *Liquid Pint* and *Gallon*, but are designated by the abbreviations (O.) (Latin *octarius*, pint), and *Cong.* (Latin *congius*, gallon). 60 ℥ = 1 f ʒ, 8 f ʒ = 1 f ℥, and 16 f ℥ = 1 (O.).

Physicians in making prescriptions frequently call a minim a *drop*, a fluid drachm a *teaspoonful*, 4 fluid drachms a *tablespoonful*, a fluid ounce 2 *tablespoonfuls*, 4 fluid ounces a *teacupful*, and a pint 4 *teacupfuls*.

These measures are very indefinite, and in fact are much in excess of what they are called. Thus a drop of most liquids is much more than a *minim*. A common teaspoon holds nearer 90 than 60 drops of water, and we more frequently find teacups that hold ½ a pint than a *gill*.

℞ is an abbreviation for *recipe*, or take; ā, aa, for equal quantities; *ss.*, for *semi*, or half; gr., for grain; *gtt.*, for drop; *P.*, for *particula*, or little part; *P. æq.*, for equal parts; *q. p.*, as much as you please.

1. How many pints in a gallon? In 5 gallons? 7 gallons? A barrel?

2. How many quarts in a barrel? In 5 bbl.? In 4 bbl.?

3. A box which would hold a cord of wood would hold how many bushels of wheat? How many barrels of water?

4. How many bushels in a barrel of 31½ gallons?

5. It is customary to heap the measure in measuring apples, potatoes, corn in the ear, ashes, and some other substances; so that about 5 pecks are sold as a bushel. According to this method of measuring, how many bushels does a barrel of 31½ gal. contain?

6. How many drops make a teaspoonful? How many teaspoonfuls make a teacupful?

7. A man sold me 32 quarts, which he called a bushel of berries; but he measured them in a liquid-quart measure, instead of a dry-quart measure. What part of a bushel did I get? How many liquid quarts should he have given me?

[For further practical applications, see *Mensuration*.]

MEASURES OF WEIGHT.

232. There are *Three Varieties* of measures of weight; viz., *Avoirdupois*,[1] *Troy*,[2] and *Apothecaries'*.

233. Avoirdupois Weight is the common weight used by grocers, and for most ordinary purposes. The denominations are *Ounces* (*oz.*), *Pounds* (*lb.*), *Hundred Weight* (*cwt.*), and *Tons* (*T.*).

234. Troy Weight is the weight used for weighing gold,

[1] From three French words, *avoir du poids*, meaning "to have weight."

[2] From *Troyes* in France, whence this weight came into use.

silver, and precious stones, and in philosophical experiments. The denominations are *Grains* (*gr.*), *Pennyweights* (*pwt.*), *Ounces* (*oz.*), and *Pounds* (*lb.*).

235. **Apothecaries' Weight** is used in medical prescriptions. The denominations are *Grains* (*gr.*), *Scruples* (℈), *Drams* (ʒ), *Ounces* (℥), and *Pounds* (*lb.*). Drugs and medicines are bought and sold in the quantity by Avoirdupois Weight.

236. ***Tables of Measures of Weight.***

AVOIRDUPOIS WEIGHT.

16 oz. = 1 lb.
100 lb. = 1 cwt.
20 cwt. = 1 T.

TROY WEIGHT.	APOTHECARIES' WEIGHT.
24 gr. = 1 pwt.	20 gr. = 1 sc., or ℈.
	3 ℈ = 1 dr., or ʒ.
20 pwt. = 1 oz.	8 ʒ = 1 oz., or ℥.

12 oz. = 1 lb., or ℔.

237. The denominations of like name in troy and apothecaries' weights are the same; but the avoirdupois pound is heavier, whereas the ounce is lighter, than the pound and ounce of the other weights. The troy pound is 22.79 cu. in. of water; and the avoirdupois pound is $\frac{175}{144}$ times as much, or 27.69 cu. in. of water. A pint of water ($28\frac{7}{8}$ cu. in.) is a little more than 1 lb. avoirdupois.

The only difference between the troy and the apothecaries' tables is in the subdivision of the ounce. In troy weight there are two subdivisions, — pennyweights and grains; whereas in apothecaries' there are three, — drams, scruples, and grains: but, in each, 480 grains make an ounce. 7000 troy grains are equal to a pound avoirdupois.

238. Physicians in writing prescriptions use the Roman notation to designate the number of grains, scruples, etc., and write them *after* the symbol. In writing thus, a final i is written j; thus gr. vj is 6 *grains,* ℈ ij is 2 scruples, ʒ iij is 3 drams, ℥ iv is 4 ounces, etc.

239. In wholesale transactions in coal and iron, and in the United-States custom-houses, 112 lb. are called a cwt.: hence 28 lb. is a *Quarter* (of a cwt.), and 2240 lb. make a *Ton*. This is sometimes called the *Long Ton*.

240. 196 lb. *flour* make a barrel. 200 lb. *pork* or *beef* make a barrel. 56 lb. *butter* = 1 firkin. 100 lb. *grain* or *flour* are called a *Cental*, of *dried fish* a *Quintal*, and of *nails* a *Keg*. 280 lb. *salt* = 1 barrel at New-York Salt-Works. A *bushel* of coal is 80 lbs.

241. ***Table showing the Weight of a Bushel of the Principal Grains and Seeds, as established by Law in the Several States.***

Barley.	Ill., Ind., Io., Ky., Mich., Minn., Mo., N. C., N. J., Ohio, Wis., **48** lb. Mass., Ore., Vt., 46 lb.; W. T., 45 lb.; La., 32 lb.; Penn., 47 lb.; Cal., 50 lb.
Buck-wheat.	Mich., Minn., Ore., Wis., 42 lb.; Io., Ill., Ky., Mo., 52 lb.; Ind., N. C., N. J., 50 lb. Cal., 40 lb.; Mass., Vt., 46 lb.; N. Y., Penn., 48 lb.; Conn., 45 lb.
Clover Seed.	Ill., Ind., Io., Ky., Mich., Minn., Mo., N. Y., Ohio, Ore., W. T., Wis., **60** lb. N. J., 64 lb.
Indian Corn.	Conn., Del., Ind., Io., Ill., Ky., La., Mass., Mich., Minn., N. J., Ohio, Ore., Penn., Vt., W. T., Wis., **56** lb. Cal., Mo., 52 lb.; N. C., 54 lb.; N. Y., 58 lb.
Oats.	Cal., Ill., Ind., La., Mich., Minn., N. Y., Ohio, Penn., Vt., Wis., **32** lb. Me., Mass., N. C., N. H., N. J., 36 lb.; Io., Mo., 35 lb.; W. T., 36 lb.; Conn., 28 lb.; Ky., 100 lb. to 3 bu.
Rye.	Conn., Ind., Io., Ill., Ky., Mass., Mich., Minn., Mo., N. J., N. Y., Ohio, Ore., Penn., Vt., W. T., Wis., **56** lb. Cal., 54 lb.; La., 52 lb.
Timothy Seed.	Ill., Ind., Io., Ky., Mo., **45** lb. N. Y., 44 lb.; Wis., 46 lb.
Wheat.	**60** lb. in all except Conn. In Conn., 56 lb.

***Peas*, *Beans*, and *Potatoes* are usually weighed at 60 lb. to the bushel.**

1. On the common grocers' scales, what part of a pound is 8 oz.? 4 oz.? 12 oz.?

2. What is the difference in weight between 3 oz. of butter and 3 oz. of gold, each being weighed by its proper weight?

3. How many bushels in a ton of wheat, Michigan standard?

4. On a road when 1 T. would be a fair load for a span of horses, how many barrels of salt ought a man to put on as a load? How many of flour?

5. How much does a barrel of water weigh, if 1 pt. weighs 1 lb.? How much a gallon?

6. In a certain hay-field there were 1250 heaps, which would average 85 lb. each. How many tons in the field?

7. How many barrels of pork will be cut from 120 hogs which average 175 lb. each?

8. How many barrels of flour will be made from 2160 bu. of wheat, if it yield 40 lb. to the bushel?

9. How many scruples in 10 drams? How many grains in ℈ vij? In ʒ v? In ℥ iij?

10. A man ordered a 3-oz. gold watch-case; but, when it came, it weighed only 57 pwt. How much did it fall short of the required weight?

11. How much will a gold watch-case weighing $2\frac{1}{2}$ oz. cost, at \$0.90 per pwt. and \$20 for making?

12. How many bushels of coal in a ton as weighed at the mines?

13. At \$6.00 per ton, what is coal per bushel?

14. The specific gravity of ice being .93 (i.e., it being .93 times as heavy as water), how many cubic feet in a ton of ice?

MEASURES OF TIME.

242. The denominations of time are *Seconds* (*sec.*), *Minutes* (*min.*), *Hours* (*hr.*), *Days* (*da.*), *Weeks* (*wk.*), *Months* (*mo.*), *Years* (*yr.*), and *Centuries* (*cen.*).

243. *Table.*

60 sec. = 1 min.	365 da. = 1 common yr.
60 min. = 1 hr.	366 da. = 1 leap yr.
24 hr. = 1 da.	100 yr. = 1 cen.
7 da. = 1 wk.	

In *Computing Interest*, 30 da. = 1 mo. For many purposes, 4 wk. are called a month.

Of the 12 calendar months which make up the year, September, April, June, and November have 30 da. each. All the others, except February, have 31 da. each. In common years February has 28 da., in leap year 29 da. What two exceptions are there to the law that each alternate month has 31 days?

244. Every year whose number is divisible by 4, except the centennial years which are not divisible by 400, is a **Leap Year.**

Thus 1840, 1844, 1880, 1876, 1600, 1200, 2000, are leap years. 1551, 1842, 1883, 1500, 1100, 1900, are not leap years.

The reason for *Leap Year* is this: A year is the time it takes the earth to go around the sun. But this is a little more than 365 days. Instead of reckoning this part of a day, it is neglected, and a *whole* day is added to the year every 4th year (in general); but, as this is a little too much, the centennial years (in general), although they are the 4th years, are reckoned as common years (365 da.). But this again is rejecting too many leap years: so that every centennial year which is divisible by 400 is made a leap year. With this correction the error does not amount to a day in 100000 years.

1. How many seconds in $\frac{1}{2}$ a minute? In $\frac{1}{4}$ of a minute? In 2 min.? In 10 min.? In $2\frac{1}{2}$ min.?

2. How many minutes in $\frac{1}{2}$ an hour? In $\frac{1}{4}$ an hour? In $\frac{3}{4}$ an hour? In $1\frac{1}{2}$ hr.? In 3 hr.? In $5\frac{1}{4}$ hr.?

3. What part of a day is 12 hr.? 6 hr.? 1 hr.? 5 hr.?

4. How many days in 48 hr.? In 36 hr.? In 80 hr.?

5. Point out the leap years in the following: 1540, 1320, 1000, 800, 560, 2100, 2290, 2400, 1875, 1876, 1877, 1878, 1879, 1880, 1881, 1882, 1883, 1890.

Important Facts concerning Dates.

245. Let the pupil verify the following: —

(*a*) A *Common Year* has 52 weeks and 1 day.

A *Leap Year* has 52 weeks and 2 days.

(*b*) *Any Particular Date*, as July 4, falls one day later in the week in a year succeeding a *Common Year*, and 2 days later in a year succeeding *Leap Year*.

(*c*) *To find from the present date the day of the week of the same day in any other month of the same year.* — Take the sum of the excesses over 28 da. of the days in each month ending in the interval. Divide this sum by 7, and, if the date be future, *advance* the day of the week by this remainder; if past, move the day of the week *back*.

(*d*) *To find from the present date the day of the week of any preceding or succeeding date.* — Reckon a day for each year closing in the interval, adding 1 da. more for each leap year. Divide the sum by 7, and, if the date be *future*, *advance* the day of the week by the remainder; if past, move the day of the week *back* by the same number. This will give the day of the week of the present date in the required year. Then pass to the required date in that year, as in the preceding.

(*e*) *To find the number of leap years that have ended in any given period.* — Divide by 4 the number of years that have ended in the time: *in general*, this quotient will be the number of leap years. But if the 1st year ending in the period is leap year, and the remainder is 1, 2, or 3; or if the 2d year is leap year, and the remainder is 2 or 3; or if the 3d year is leap year, and the remainder is 3, — add 1 to the quotient. Finally, reject 1 for each centennial year not divisible by 400.

(*f*) It of course amounts to the same thing to advance the date of a particular day of the week as to keep the same date and throw the day of the week back, etc. This is often more convenient in practice.

Ex. 1. — To-day (April 16, 1879) is Wednesday. What day of the week was July 24, 1827?

Between dates 52 years have ended, 13 of which were leap years. $(52 + 13) \div 7 = 9$ and 2 remainder. Hence April 16, 1827, was Monday. Now, the excesses of 28 in the months ending between this and July 24 is $2 + 3 + 2 = 7$. Hence the days of the week fall on the same days of the month in July as in April. July 16, 1827, was therefore Monday, and July 24 Tuesday.

2. Find from your present date that July 4, 1776, was Wednesday.

3. On what day of the week were you born?

4. On what day of the week will next Christmas fall? The 4th of July in the year 2000? (Find these from your present date.)

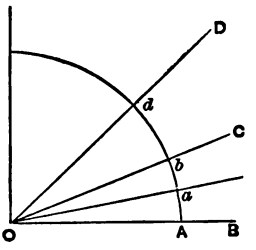
5. Show that the days of the week fall on the same days

of the month in January as in October, in March as in November, in April as in July, in September as in December.

6. If May, June, or Aug. 1, of a common year, falls on Sunday, show that no succeeding month of that year begins on Sunday. How is this in leap year?

CIRCULAR OR ANGULAR MEASURE.

246. A Degree is $\frac{1}{360}$ part of the circumference of a circle, or an angle measured by this part of a circumference, and is designated thus, 1°.

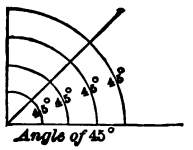


In the figure from **A** to a is 10 degrees, written 10°. From **A** to b is 20°, and the angle **BOC** is an angle of 20°. From **A** to d is 45°, and the angle **BOD** is an angle of 45°. A quarter of a circumference, or a right angle, is 90°.

247. A Minute (as circular or angular measure) is $\frac{1}{60}$ part of a degree, and is designated thus, 1′. 5 minutes of this measure is written 5′.

248. A Second (as circular or angular measure) is $\frac{1}{60}$ part of a minute, and is written 1″. 12 seconds is written 12″.

It will be observed that a degree, *circular* measure, is not a definite length: thus a degree on the circumference of the earth is about 69½ miles, while a degree on such a circle as the one in the last figure is so small that we could not see it. But a degree as the measure of an angle is a definite thing. Thus, if we draw an angle equal to $\frac{1}{4}$ a right angle, and then strike a circumference with the vertex of the angle as a centre, there will be 45° of the circumference included between its sides, whether the circle be large or small.



249. ***Table of Circular or Angular Measure.***

$60'' = 1'$, $60' = 1°$, $90° = 1$ Quadrant or Right Angle.

250. A *Geographic* or *Nautical* (marine) mile is $1'$ of the equator, or about $1\frac{1}{6}$ common miles.

251. Degrees of Latitude being distances north or south of the equator, and measured on equal circles (great circles which pass through the poles), are nearly equal; but degrees of Longitude vary from $69\frac{1}{2}$ miles at the equator to 0 at the poles. In the latitude of Ann Arbor, Mich., a degree of longitude is 51.1 miles. At 60° latitude, a degree of longitude is 34.53 miles.

1. How many minutes in 2°? In $1\frac{1}{3}°$? In 10°? In .5 of a degree? In .3?
2. How many degrees in 240′? In 580′? In 200′? In 1260′ how many degrees and decimals of a degree?
3. How many seconds in $\frac{1}{8}$ a minute? In .75 of a minute? In .25 of a minute?
4. On a circumference which is 1800 ft. in length (around), how long is a degree? A minute? A second?
5. The circumference of a circle is 3.1416 times its diameter (nearly). Show that a degree on the circumference of the earth is about $69\frac{1}{2}$ miles, the radius of the earth being 3962 miles.

252. Of Paper 24 *Sheets* are called a *Quire*; 20 *Quires*, a *Ream*; 2 *Reams*, a *Bundle*; 5 *Bundles*, a *Bale*.

253. In Counting, 12 *Things* are called a *Dozen*; 12 *Dozens*, a *Gross*; 12 *Gross*, a *Great Gross*; 20 *Things*, a *Score*.

254. Two things are often called a *pair*, and six things a *set*; as a *pair* of birds, a *set* of spoons.

255. The terms *folio*, *quarto*, *octavo*, applied to books, indicate the number of leaves into which a sheet of paper is folded. Thus, when a sheet of paper is folded into 2, 4, 8, 12, 16, 18, or 24 leaves, the forms are called respectively, folio, 4to or quarto, 8vo or octavo, 12mo, 16mo, *18mo*, and 24mo.

LONGITUDE AND TIME transferred to page 203.

SECTION II.

REDUCTION.

256. Reduction of Denominate Numbers is the process of changing the denomination in which a number is expressed without altering the value represented by it. If the change is from higher denominations to lower, the reduction is said to be *Descending;* if from lower to higher, *Ascending.*

1. In 4 yd. 2 ft. 9 in. how many inches?

OPERATION.	EXPLANATION.
4 yd. 2 ft. 9 in. 3 14 ft. 12 177 in.	Since 3 ft. make a yard, in any number of yards there are 3 times as many feet as yards. Hence in 4 yd. there are 3 times 4, or 12 ft.; which, with the 2 ft., make 14 ft.

Since 12 in. make a foot, in any number of feet there are 12 times as many inches as feet. Hence in 14 ft. there are 12 times 14, or 168 in.; which, with the 9 in., make 177 in. Hence 4 yd. 2 ft. 9 in. are 177 in.

Is this reduction descending, or reduction ascending? Why?

2. In 5127 pt. of water, how many barrels, gallons, quarts, and pints? Or reduce 5127 pt. liquid measure to the higher denominations of that measure.

OPERATION.	EXPLANATION.
2) 5127 pt. 4) 2563 qt. 1 pt. 640 gal. 3 qt. 31.5) 640.0 (20 bbl. 630 10.0 gal.	Since every 2 pt. make a quart, in 5127 pt. there are as many quarts as 2 is contained times in 5127; i.e., 2563 qt. and 1 pt. [Let the pupil fill out the explanation.] ∴ 5127 pt. = 20 bbl. 10 gal. 3 qt. 1 pt.

Is this reduction descending, or reduction ascending? Why?

To Reduce Denominate Numbers.

257. **Rule.** — I. **Reduction Descending** *is performed by beginning with the highest denomination given, multiplying this by the number which it takes of the next lower to make one of this higher, and adding to this product the given number of this next lower denomination. Reduce this sum to the next lower denomination in a like manner, and add to the result the given number of this lower. Proceed in this manner till the denomination required is reached.*

II. **Reduction Ascending** *is performed by dividing the given number by the number of this denomination which it takes to make one of the next higher denomination, and treating the quotient thus arising in like manner, proceeding thus till the desired denomination is reached.*[1]

Examples for Practice.

Each of the following will afford an exercise for both forms of reduction: —

1. 3 da. 5 hr. 17 min. = 278220 sec.[2]
2. 442 pt. = 6 bu. 3 pk. 5 qt.
3. 6 gal. 3 qt. 1 pt. = 55 pt.
4. £32 12*s.* 10*d.* = 7834*d.*
5. 8 lb. 6 oz. 12 pwt. 16 gr. = 49264 gr.
6. ℥ vj ʒ iij ℈ j = 154 sc.

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[1] It is scarcely expedient to give an abstract demonstration of a rule so mechanical as this. In fact, the author has doubts about the expediency of giving the *rule* at all. Be careful, however, that the *rationale* of the process is understood, and can be given with ease and elegance.

[2] These examples should be assigned in both ways, each making two examples, — one in reduction descending, and one in reduction ascending. Thus this one will be given, "Express 3 da. 5 hr. 17 min. in seconds;" and again, "Express 278220 sec. in days, hours, and minutes." In assigning them for work on the blackboard the two examples may be assigned to two students standing together, and thus one will "prove" the other's work.

7. 78692 gr. = 13 lb. 7 oz. 18 pwt. 20 gr.
8. 5382 oz. (avoirdupois) = 3 cwt. 36 lb. 6 oz.
9. 78562 cu. ft. = 613 cd. 98 cu. ft.
10. 4268 sq. rd. = 26.675 A.
11. $\frac{3}{4}$ of a bbl. = 189 pt.

SUGGESTION. — This should be solved thus:—

$$\frac{3}{4} \times \frac{63}{2} \times 4 \times 2 = 189.$$

Why is $\frac{3}{4}$ multiplied by $\frac{63}{2}$? Is this quite in accordance with the rule (**257**)?

The converse process is thus performed:—

$$\frac{189}{2\times4} \div 31\tfrac{1}{2} \text{ is } \frac{189}{4\times2} \times \frac{2}{63} = \frac{3}{4}.$$

Does this process differ from that given in the rule (**262**)?[1]

12. $4\frac{3}{4}$ cords = 608 cu. ft.
13. $\frac{2}{3}$ gal. = $5\frac{1}{3}$ pt. $\frac{1}{2}$ pt. = $\frac{1}{16}$ gal.
14. 2 A. 110 sq. rd. = 430 sq. rd.
15. 6875988 cu. in. = 31 cd. 11 cu. ft. 276 cu. in.
16. 12 bbl. 19 gal. 2 qt. = 3180 pt.
17. 368 pt. = 5 bu. 3 pk.
18. 30630 min. = 21 da. 6 hr. 30 min.
19. 172800″ = 48°.
20. 17 sq. ft. 27 sq. in. = 2475 sq. in.
21. 75288 gr. = 13 lb. 17 pwt.
22. 3.25 gal. = 26 pt.

Does the process by which we perform this constitute an exception to the general rule (**257**)?

23. 12 ft. 9 in. = 12.75 ft.

How are inches reduced to feet? 9 ÷ 12 = what decimal? Does this process constitute an exception to the rule (**257**)?

24. 3 in. = .083$\frac{1}{3}$ yd.

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[1] It should be the aim of the teacher to show that we need no *special* rules for reduction of denominate fractions.

How are inches reduced to feet? How are feet reduced to yards? $3 \div 12 = .25$. $.25 \div 3 = .083\frac{1}{3}$.

25. £4.67 = £4 13*s*. 4*d*. 3.2 *far*.

How is the £ .67 reduced to shillings? How is .4*s*. reduced to pence?

26. $\frac{7}{30}$ wk. = 1 da. 15 hr. 12 min.

$\frac{7}{30} \times 7 = 1\frac{1}{30}$. $\frac{1}{30} \times \overset{4}{24} = 15\frac{1}{5}$. $\frac{1}{5} \times 60 = 12$.

$\therefore$ $\frac{7}{30}$ wk. = 1 da. 15 hr. 12 min.

$12 \div 60 = \frac{1}{5}$. $15\frac{1}{5} \div 24 = \frac{\overset{19}{76}}{5} \div \overset{6}{24} = \frac{19}{30}$. $1\frac{19}{30} \div 7 = \frac{49}{30} \div 7 = \frac{7}{30}$.

N.B.—Let the pupil trace this work till he sees clearly that the processes form no exceptions to the general rules for reduction.

27. $.475° = 28' 30''$.

28. ʒ iv ℈ ij = $\frac{7}{12}$ oz.

29. .345 of a bbl. of flour = 67 lb. $9\frac{23}{25}$ oz.

30. 16 lb. 8 oz. = .275 bu. of wheat, or $\frac{11}{40}$ bu.

31. 43 rd. 11.7 in. = .13456 mi.+.

32. $\frac{5}{6}$ oz. = 16 pwt. 16 gr.

33. $\frac{4}{5}$ bu. = 3 pk. 1 qt. $1\frac{1}{5}$ pt.

34. 213 rd. 1 yd. $2\frac{1}{2}$ ft. = $\frac{2}{3}$ mi.

35. A pile of wood 4 ft. long, 3 ft. high, 4 ft. wide = $\frac{3}{8}$ cd.

36. ℥ ix ʒ j ℈ ij gr. viij = .76875 lb.

37. 3 lb. 10 oz. troy = 3 lb. $2\frac{1}{2}$ oz. avoirdupois nearly.

38. 528 chains = 6 mi. 192 rd.

39. $2\frac{3}{4}$ mi. = 220 ch.

40. 3 ch. 4 lk. = 200.64 ft.

41. $2\frac{1}{2}$ A. = 25 sq. ch.

42. A lot 12 ch. 24 lk. by 15 ch. 40 lk. = 18.8496 A.

$12.24 \times 15.4 \div 10 = 18.8496$.

43. 3 A. = 300000 sq. lk.
44. A lot 81 ch. 52 lk. by 34 ch. 2 lk. = 277.3 A.+.
45. 756 cu. ft. = 1306368 cu. in., or $5.9\frac{1}{16}$ cd.
46. .24 bu. wheat = 14 lb. 6.4 oz.
47. 75 sq. rd. 12 sq. yd. 4 sq. ft. = $20530\frac{3}{4}$ sq. ft.
48. 165888 cu. in. = $\frac{3}{4}$. cd.
49. 8410 lk. = 1 mi. 4.1 ch., or 1.05125 mi.
50. $24\frac{1}{2}$ lb. = $\frac{1}{8}$ bbl. flour.
51. 5 f℥ 36 ℔ = .7 f℥.
52. .008 mi. = 2 rd. 9.24 ft.
53. \$68 = 680 d. = 6800 ct. = 68000 m.
54. 3256 m. = 325.6 ct. = 32 d. 5 c. 6 m. = \$3.256.
55. 1250^{fr} = \$241.25. \$10 = 51^{fr} 81^{c}+.
56. 5^{Nap} = \$19.30. \$100 = 25^{Nap} 18^{fr} 13^{c}+.
57. 5 fathoms = —— ft.? 256 ft. = —— fathoms?
58. 2 mi. 34 rd. 3 yd. = —— yd.?
59. 200 yd. = —— mi.?
60. 1 mi. = yd.? 40 rd. = —— yd.?
61. 100 yd. = —— rd.? $\frac{2}{3}$ yd. = what part of a rd.?
62. A lot 5 rd. by 8 = what part of an acre?
63. $3\frac{3}{4}$ A. = —— sq. yd.? 5 sq. rd. = —— A.?
64. A lot 7.21 ch. by 3.40 ch. = 2.4514 A.
65. 5 A. 110 sq. rd. 1 sq. yd. = 5.6877 A. very nearly.
66. $\frac{2}{5}$ cd. = —— cu. ft.? 12 cu. ft. = what part of a cd.?
67. 5 cd. 120 cu. ft. = —— cu. yd.?
68. 1376 cu. yd. = $290\frac{1}{4}$ cd. 1 cu. yd. = —— cd.?
69. 3 bbl. 20 gal. = —— qt.?
70. 4000 qt. = 31 bbl. $23\frac{1}{2}$ gal. = $31\frac{47}{63}$ bbl.
71. 10 gal. 3 qt. 1 pt. = 43.5 qt.
72. $126\frac{3}{4}$ qt. = 31 gal. 2 qt. $1\frac{1}{2}$ pt. = 1 bbl. $1\frac{1}{2}$ pt.
73. 43 qt. = 5 pk. 3 qt., or $5\frac{3}{4}$ pk.
74. $\frac{1}{12}$ bu. = $5\frac{1}{3}$ pt.
75. 3.416 bu. = 3 bu. 1 pk. $5\frac{1}{3}$ qt. nearly.
76. 2 bu. 3 pk. 5 qt. $1\frac{1}{2}$ pt. = 187.5 pt.

77. 1 bu. 1 pk. 1 qt. 1 pt. = how many pecks?
78. 2 bbl. 15 gal. $110\frac{1}{4}$ pt. = 91.78125 gal.
79. 5287 qt. = —— bbl. —— gal. —— qt. —— pt.?
80. 786 gal. = —— pt.?
81. 4 lb. 8 oz. 12 pwt. 16 gr. = 27184 gr.
82. 3 lb. 10 oz. 20 gr. = $46\frac{1}{24}$ oz.
83. $342\frac{2}{5}$ oz. = 28 lb. 6 oz. 8 pwt.
84. 1 T. 13 cwt. 58 lb. = 3358 lb.
85. 7129 lb. = 3 T. 11 cwt. 29 lb.
86. 7129 lb. = 3 T. 3 cwt. 73 lb. U. S. customs weight.
87. 52 rd. 4 yd. 2 ft. = $\frac{1983}{6600}$ mi.
88. $1\frac{1}{12}$ bbl. = 34 gal. 1 pt.
89. 2 hr. 52. min. 48 sec. = $\frac{2}{25}$ da.
90. $\frac{5}{8816}$ bu. = $\frac{5}{14}$ pt.
91. 11 da. = $\frac{11}{30}$ mo. = $\frac{11}{360}$ yr. in computing interest.
92. 2 mo. 13 da. = .2028— yr. in computing interest.
93. 2 yr. 5 mo. 7 da. = 2.436 yr.—, as above.
94. 11 yr. 10 mo. 21 da. = 11.89 yr.+, as above.
95. 4 yr. 10 mo. 24 da. = 58.8 mo., as above.
96. 3.725 yr. = 3 yr. 8 mo. 21 da., as above.
97. 7 mo. 6 da. = .6 yr., as above.
98. 1 yr. 8 mo. 12 da. = 1.7 yr., as above.
99. 1 yr. 4 mo. 24 da. = 1.4 yr., as above.

SECTION III.

ADDITION.

1. There are three casks which contain 2 gal. 3 qt. 1 pt., 5 gal. 2 qt. $1\frac{1}{2}$ pt., and 4 gal. 1 qt. $1\frac{1}{2}$ pt. How much do they all contain?

How many pints are $1\frac{1}{2}$, $1\frac{1}{2}$, and 1? How many quarts does this make? 2 qt., 1 qt., 2 qt., and 3 qt., make how many quarts? How *many* gallons does this make, and how many quarts over? 2 gal., *4 gal., 5 gal., and 2 gal.*, make how many gallons?

2. There are 5 pieces of rope whose respective lengths are 2 yd. 2 ft. 3 in., 4 yd. 1 ft. 7 in., 3 yd. 2 ft. 5 in., 5 yd. 2 ft. 10 in., and 3 yd. 2 ft. What is the entire length?

Operation.	Explanation.
2 yd. 2 ft. 3 in. 4 yd. 1 ft. 7 in. 3 yd. 2 ft. 5 in. 5 yd. 2 ft. 10 in. 3 yd. 2 ft. 20 yd. 2 ft. 1 in.	We write numbers of the same denomination in the same column, because such are more conveniently added together. We then begin the addition with the *lowest* denomination, because we can thus tell whether there will arise any of the higher denominations from adding the lower, and, if there does, can add it with the higher denominations as we go along. [Were we to *commence* with the highest denomination, we should have to revise our results after having added all the columns. Let the pupil try it.]

In this example the sum of the inches column is 25 in. = 2 ft. 1 in. The sum of the feet column, together with the 2 ft. from the inches column, is 11 ft. = 3 yd. 2 ft. The sum of the yards column, together with the 3 yd. from the feet column, is 20 yd.

To Add Compound Numbers.

258. Rule. — *Write the numbers so that like denominations shall stand in the same column.*

Beginning with the lowest denomination, add the numbers in it, and divide the sum by the number it takes of this denomination to make 1 of the next higher. Write the remainder under the column added, and add the quotient to the next denomination.

Proceed in this manner with each denomination in succession, writing the entire sum of the highest.

The pupil should give the reasons (Demonstration) in order. He is to tell (1) Why we write the numbers as the rule directs; (2) Why we begin with the lowest denomination; (3) Why we reduce the several sums to the higher denominations. (See "Explanation" above.)

3. What is the amount of £105 1s. 2d. 3 far., £218 11s. 5d. 2 far., £199 17s. 9d. 2 far., and £77 18s. 3d. 3 far.?

4. What is the sum of 8 lb. ℥ xj ʒ vj ℈ ij, 9 lb. ℥ x ʒ vij ℈ j, 4 lb. ℥ vij ʒ iij ℈ j, 17 lb. ℥ viij ʒ iij ℈ j, and 45 lb. ℥ xj ʒ iij ℈ j?

5. What is the sum of 10 rd. 3 yd. 1 ft. 7 in., 7 rd. 2 yd. 2 ft. 5 in., 3 rd. 4 yd. 1 ft. 9 in., 5 rd. 2 yd. 1 ft. 10 in., and 13 rd. 4 yd. 11 in.?

6. What is the sum of 145 bu. 3 pk. 1 qt., 163 bu. 1 pk. 3 qt., 275 bu. 2 pk. 7 qt., 45 bu. 3 pk. 6 qt., and 73 bu. 1 pk. 5 qt.?

7. What is the amount of £13 17s. 11d. 1 far., £22 14s. 9d. 1 far., £37 18s. 6d. 3 far., and £46 13s. 7d. 2 far.?

8. A silversmith bought of A 3 lb. 9 oz. 14 pwt. 16 gr. of silver, of B 9 lb. 11 oz. 17 pwt. 18 gr., of C 1 lb. 8 oz. 19 pwt. 21 gr., and of D 3 lb. 7 oz. 12 pwt. 16 gr. How much silver did he buy?

9. What is the amount of 45 cd. 23 cu. ft. 25 cu. in., 273 cd. 75 cu. ft. 684 cu. in., 97 cd. 18 cu. ft. 384 cu. in., 250 cd. 64 cu. ft. 197 cu. in., and 264 cd. 84 cu. ft. 848 cu. in.?

10. What is the sum of 86 sq. yd. 7 sq. ft. 46 sq. in., 245 sq. yd. 8 sq. ft. 89 sq. in., and 265 sq. yd. 7 sq. ft. 128 sq. in.?

11. What is the amount of 5 bbl. 20 gal. 3 qt. $1\frac{1}{4}$ pt., 7 bbl. 25 gal. 2 qt., 28 gal. 1 qt., 3 bbl. 30 gal. $1\frac{3}{4}$ pt., 2 qt. $1\frac{1}{2}$ pt., 13 gal., 2 qt. $1\frac{3}{4}$ pt.?

12. What is the sum of $\frac{3}{4}$ of a bushel, 2.64 pecks, .5 bu., $12\frac{3}{5}$ qt., $10\frac{1}{2}$ pk., and 320 pt. in bushels?

13. What is the sum of 8 yd. 2 ft., $75\frac{1}{2}$ ft., 2005 in., 4.35 yd., $28\frac{2}{5}$ ft., $3\frac{2}{3}$ yd., .25 ft., and 226 ft. in rods?

14. Mr. E. Jones owned $1\frac{1}{2}$ sections of land in one township, 80 acres in another, a quarter section in another, a 40-acre lot in another, and a piece of land 40 rods by 40 chains in another. How much land had he in all?

SECTION IV.

SUBTRACTION.

1. From 3 lb. 8 oz. 16 pwt. subtract 1 lb. 3 oz. 12 pwt. 17 gr.

Operation.	Explanation.
3 lb. 8 oz. 16 pwt. 1 lb. 3 oz. 12 pwt. 17 gr. 2 lb. 5 oz. 3 pwt. 7 gr.	We write the denominations of the subtrahend under like denominations in the minuend, because it is more convenient to subtract a number of any denomination from another of like denomination. We begin to subtract at the lowest denomination; so that, if there should chance not to be as many of any particular denomination in the minuend as in the subtrahend, we can take one from the next higher denomination in the minuend, and put it with the number in this deficient denomination. Thus in this example there are *no* grains represented in the minuend; but we can take one of the 16 pwt. which makes 24 gr., and, subtracting 17 gr. from it, have 7 gr. left. Then 12 pwt. from 15 pwt. leaves 3 pwt., etc.

To Subtract Compound Numbers.

259. Rule. — *Write the subtrahend under the minuend so that its denominations shall fall under the corresponding denominations of the minuend.*

Begin with the lowest denomination, and take the number represented in each denomination of the subtrahend from the number in the corresponding denomination in the minuend, and write the remainder underneath.

If the number in any denomination in the minuend is less than the corresponding number in the subtrahend, take 1 of the next higher denomination of the minuend in which there are any, and reducing it to this lower denomination, and uniting it with what there may be in this denomination, perform the subtraction.

Observe, when passing to the higher denominations, how much remains in them.

2. From 14 bu. 3 pk. 4 qt. 1 pt. subtract 8 bu. 3 pk. 7 qt.
3. From £10 12*s.* 7*d.* take £6 8*s.* 5*d.*
4. From £17 0*s.* 3*d.* take £9 10*s.* 5*d.*

Operation.	Explanation.
£17 0*s.* 3*d.* £ 9 10*s.* 5*d.* £ 7 9*s.* 10*d.*	As 5*d.* cannot be taken from 3*d.*, we take £1 = 20*s.*, and taking 1 of the 20*s.*, which makes 12*d.*, subtract 5*d.* from 12 + 3*d.*, or 15*d.* Then we have 10*s.*, which we subtract from the 19*s.* remaining of the £1. Finally we take £9 from £16.

5. From 10 bu. 3 pk. 4 qt. take 4 bu. 1 pk. 2 qt.
6. From 1 bu. 1 pk. 1 qt. take 2 pk. 1 qt. 1 pt.
7. From $\frac{1}{2}$ bu. take 3 qt. and 2 pt.
8. From 5 bbl. 24 gal. 2 qt. take 1 bbl. 27 gal. 3 qt.
9. From 4 lb. 8 oz. 12 pwt. take 2 lb. 5 oz. 17 gr.
10. From 1 lb. take 15 pwt. 20 gr.
11. From 5 mi. 100 rd. 12 ft. take 2 mi. 30 rd. 15 ft.
12. From 27 A. take 110$\frac{1}{4}$ sq. rd.
13. From 2$\frac{1}{2}$ cords take 120.32 cu. ft.
14. From 5 yd. take 5 ft.
15. From 2 T. 12 cwt. take 3420 lb.
16. From $\frac{1}{2}$ section take 51 A. 45 sq. rd.
17. From ℥ vij ʒ iij ℈ j take ℥ iij ʒ v ℈ ij.
18. Sold from a barrel of molasses 10 gal. 3 qt. 1 pt. How much remained?
19. From the sum of 3 lb. 13 oz., 2 lb. 5 oz., and 6 lb. 11 oz., take 12 lb. 10 oz.
20. From $\frac{1}{4}$ yd. take 7 in.

To Find the Time between Two Dates.

260. There are three methods in use for estimating the time between two dates; viz.: —

(1) To find the exact time in days.

(2) To find the time as commonly reckoned in computing interest; that is, reckoning 30 days as a month, and 12 months as a year.

(3) To find the time in calendar years, months, and days.

261. EXACT TIME IN DAYS.

1. What is the *exact time in days* between Jan. 12, 1856, and May 15, 1874?

SOLUTION. — Between Jan. 12, 1856, and Jan. 12, 1874, are $1874 - 1856 = 18$ yr., in which there are 5 Februaries containing 29 days each.[1] Hence in these 18 yr. there are $365 \times 18 + 5 = 6575$ days. Then from Jan. 12 to May 15, 1874, there are $19 + 28 + 31 + 30 + 15 = 123$ days. Therefore there are in all $6575 + 123 = 6698$ days between these dates.

2. What is the *exact time in days* between April 5, 1860, and Nov. 24, 1875?

3. Find the *exact time in days* between the 4th of July and Christmas.

4. Between May 6, 1878, and Aug. 10, 1878.

5. Between July 17, 1875, and April 11, 1876.

6. Between Oct. 7, 1878, and Jan. 23, 1879.

7. Between March 21, 1879, and May 18, 1879.

8. Between June 7, 1876, and Aug. 15, 1876.

9. Between Jan. 5, 1872, and April 1, 1872.

10. Between Jan. 5, 1873, and April 1, 1873.

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262. TO FIND THE TIME BETWEEN TWO DATES, RECKONING 30 DA. = 1 MO., AND 12 MO. = 1 YR.

1. Reckoning 30 da. a month, and 12 mo. a year, what is the time between Aug. 17, 1871, and Feb. 3, 1875?

[1] That is, *those* in the leap years 1856, 1860, 1864, 1868, and 1872.

Operation.	Explanation.
1875 yr. 2 mo. 3 da. 1871 yr. 8 mo. 17 da. 3 yr. 5 mo. 16 da.	The later date (larger number) is the 1875th year, 2d month, 3d day. The earlier is the 1871st year, 8th month, and 17th day. These dates are subtracted as in ordinary subtraction, calling 30 days a month, and 12 months a year.[1]

Find the time between the following dates, reckoning 12 mo. a year, and 30 da. a month: —

2. March 16, 1850, and Dec. 5, 1871.
3. Aug. 23, 1846, and April 2, 1827.
4. Dec. 22, 1620, and Jan. 19, 1875.
5. May 5, 1872, and July 15, 1873.
6. April 17, 1878, and July 7, 1880.
7. Oct. 9, 1872, and May 30, 1876.
8. Dec. 27, 1874, and June 30, 1878.

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263. To find the Time, reckoning Calendar Months and Years.

1. Mrs. J. was born Dec. 5, 1825. What was her exact age, in calendar years, months, and days, on the 20th day of January, 1875?

Suggestion. — Such a question should be answered by giving the entire years and *calendar* months between the dates, and adding the remaining days. Thus, in this case, from Dec. 5, 1825, to Dec. 5, 1874, is 49 years; from Dec. 5, 1874, to Jan. 5, 1875, is 1 mo.; and from Jan. 5 to Jan. 20 is 15 days. Hence Mrs. J.'s age is 49 yr. 1 mo. 15 da.

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[1] A full explanation of this process requires that we understand that the later date is really 1874 yr. 1 mo. 3 da. after the Christian era, and the earlier 1870 yr. 7 mo. 17 da. after the same time. Hence we should have

1874 yr. 1 mo. 3 da.	
1870 yr. 7 mo. 17 da.	
3 yr. 5 mo. 16 da.	

But it is a little more convenient to take the dates as we are accustomed to name them, and subtract. That this process gives the correct difference is evident, since it is only increasing both minuend and subtrahend by 1 yr. 1 mo., and hence leaves the *difference the same.*

2. What is Mr. A.'s age June 15, 1875, he having been born Oct. 20, 1829.

All such problems can be solved by subtraction if we observe the following —

264. Rule. — In reckoning calendar months and exact days, when subtracting the days, "borrow," and, when adding, "carry," *according to the number of days which make the month next preceding that in which the period terminates;* and when obtaining *a date* by subtraction, should a 0 fall in months order, transfer 1 yr. = 12 mo. to that order.

3. What is Mr. O.'s age March 12, 1875, he having been born July 24, 1827?

1875	3	12
1827	7	24
47	7	16

The month "borrowed" is February, 28 da.

4. What is Mary's age Aug. 15, 1875, she having been born July 20, 1868?

The month "borrowed" is July, 31 da.

5. A note dated July 25, 1857, matures in 5 yr. 3 mo. 24 da. When is it due?

The month "carried" (filled out) is the 10th, 31 da.

6. June 8, 1877, I have a note which has run 2 yr. 5 mo. 10 da. What is its date?

Subtracting, we have 1875 0 29, which is Dec. 29, 1874.

7. Feb. 12, 1876, Henry is 10 yr. 1 mo. 15 da. old. When was he born?

8. What is the exact time in days from Sept. 7, 1873, to Dec. 10, 1875?

Subtracting, we have 2 yr. 3 mo. 3 da. The 2 yr. = 730 da., and the 3 mo. = 91 da.

SECTION V.

265. MULTIPLICATION.

1. Multiply 5 yd. 2 ft. 8 in. by 7.

OPERATION.	EXPLANATION.
5 yd. 2 ft. 8 in. 7 41 yd. 0 ft. 8 in.	The multiplier is written under the lowest denomination of the multiplicand as matter of custom. We commence the multiplication with the lowest denomination, since by so doing we can find how many of the next higher denomination any particular product makes, and thus add it in with the next product as we pass along. Thus 7 times 8 in. are 56 in. = 4 ft. 8 in. Writing the 8 in. under inches, we reserve the 4 ft. to be added to the next product. 7 times 2 ft. are 14 ft., which, with the 4 ft. from the preceding product, make 18 ft. = 6 yd. 0 ft. Finally, 7 times 5 yd. = 35 yd., which, with the 6 yd. from the preceding product, makes 41 yd. Hence 7 times 5 yd. 2 ft. 8 in. are 41 yd. 8 in.

2. Multiply 5 lb. 13 oz. by 8.

(1) Where do you write the multiplier? Why?

(2) Where do you begin to multiply? Why?

(3) What do you do with the product arising from multiplying any particular denomination by the multiplier? Why?

Pupil *write* the rule, and the reasons for it; that is, the *Rule* and the *Demonstration*.

3. Multiply £7 9*s*. 5*d*. by 6.

4. Multiply 1 cd. 112 cu. ft. by 10.

5. Multiply 12 gal. 3 qt. 1 pt. by 7. By 100.

6. Multiply 4 oz. 12 pwt. 21 gr. by 16. By 132.

	4 oz.	12 pwt.	21 gr.	
			132	
	9	5	18	Product by 2.
11	7	6	6	" " 30.
38	8	7	12	" " 100.
	51 lb.	0 oz.	19 pwt. 12 gr.	Product by 132.

7. Multiply 17 A. 35 sq. rd. 4 sq. yd. 5 sq. ft. by 23.

8. Multiply 4 mi. 110 rd. 17 ft. by 127. By 250.

9. Multiply 5 da. 15 hr. 13 min. 20 sec. by 341.

10. Multiply 10° 12′ 14″ by 45. By 6. By 13. By 10.
11. Multiply 3 lb. ℥ viij ʒ ij ℈ j by 4. By 12. By 24.

SECTION VI.

DIVISION.

1. Divide 23 da. 15 hr. 51 min. by 7.

OPERATION.

7) 23 da. 15 hr. 51 min.

3 da. 9 hr. 7 min. $17\frac{1}{7}$ sec.

EXPLANATION.

We begin the division with the *highest* denomination, since, if there is any remainder, it can be reduced to the next lower denomination, combined with what is expressed in this denomination, and the whole sum divided at once. Thus 23 da. ÷ 7 = 3 da. and 2 da. (= 48 hr.) remainder. Then 48 hr. + 15 hr. = 63 hr., which, divided by 7, gives 9 hr. 51 min. ÷ 7 = 7 min. and 2 min. (or 120 sec.) remainder. 120 sec. ÷ 7 = $17\frac{1}{7}$ sec.

2. Divide 12 lb. 15 oz. by 8.
3. Divide 125 bbl. 17 gal. 3 qt. by 36.

OPERATION.

36) 125 bbl. 17 gal. 3 qt. (3 bbl. 15 gal. $1\frac{7}{9}$ qt.
108
17 bbl. *remaining.*
$31\frac{1}{2}$
$8\frac{1}{2}$
17
51
17
$552\frac{1}{2}$ gal.
36
192
180
$12\frac{1}{2}$ gal. *rem.*
4
50 qt.
3 qt.
53 qt.
36
17 *qt. rem.*

EXPLANATION.

Dividing 125 bbl. by 36, we find the quotient 3 bbl., and a remainder 17 bbl. Reducing this to gallons, and adding the 17 gal. in the dividend, we have $552\frac{1}{2}$ gal. This, divided by 36, gives a quotient 15 gal., and a remainder $12\frac{1}{2}$ gal. This, reduced to quarts, makes, with the 3 qt. of the dividend, 53 qt. Dividing this by 36, we have $1\frac{17}{9}$ qt.

To Divide Compound Numbers.

266. Rule. — I. *Write the divisor on the left hand of the dividend, and the quotient underneath the dividend, or at its right, according as you divide by short or long division.*

II. *Beginning with the highest denomination, divide it, and write the quotient of this denomination in its place. Reduce the remainder (if any) to the next lower denomination, and add to it the number of this denomination in the dividend. Divide as before, reducing the remainder to the next lower denomination.*

Proceed in this manner till the division is complete.

DEMONSTRATION. — The general principle involved in this operation is the same as that involved in simple division; viz., the quotient is found by dividing the parts of the dividend separately, and adding the quotients. (See rule for Simple Division and its demonstration.)

The relative position of dividend, divisor, and quotient, is mainly matter of custom or convenience.

The division is commenced at the left hand, in order that the several remainders which may arise may be reduced and combined with the lower denominations as the work proceeds.

4. Divide £25 10*s.* 8*d.* by 9.
5. Divide 28 lb. 14 oz. by 5.
6. Divide 3 lb. ℥ vij ʒ iv ℈ ij by 6.
7. Divide 20 A. 100 sq. rd. by 10. By 27. By 13.
8. Divide 6 mi. by 115.
9. Divide 1 bbl. by 22.
10. Divide ½ a bushel by 7.
11. Divide 7¾ cords by 3.
12. Divide 5 yd. 1 ft. 8 in. by 4.

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13. How many times is 2 bu. 3 pk. 7 qt. contained in 17 bu. 2 pk.?

SUGGESTION. — Reduce both dividend and divisor to the lowest *denomination* in either, and then divide.

14. Divide 3 bbl. by 6 qt. By 20 gal. 2 qt. By 5 gal. 3 qt. 1 pt.

LONGITUDE AND TIME.

267. As the sun appears to pass around the earth from east to west, and as the hours of the day are determined by its apparent position, any given hour comes to a place at the east *before* it does to a place at the west: thus it is noon at New York *before* it is noon at Detroit, Mich. Hence, when it is noon at Detroit, it is *after* noon at New York. In like manner, when it is noon at Detroit, it is *before* noon at Omaha, Neb.

268. Principle.—*When it is any given hour of the day at any given place, it is Later at places Eastward, and Earlier at places Westward.*

269. To ascertain just how great this *Difference in Time* is, we have only to consider, that, as the sun appears to go around the earth—that is, to pass over 360° of longitude—in 24 hours, in 1 hr. it passes over 360 ÷ 24, or 15° of longitude. Then 1° of longitude makes a difference of $\frac{1}{15}$ of 1 hr. (60 min.), or 4 min. in time; and 1′ of longitude makes a difference of $\frac{1}{60}$ of 4 min. (240 sec.), or 4 sec. in time.

[NOTE.—Degrees in longitude vary[1] in length according to the latitude. See Art. **251.** At lat. 0°, a degree of longitude is $\frac{7925.64 \times 3.1416}{360}$ (see **442**), or 69⅛ mi., very nearly. In lat. 10°, 1° of long. = 68.1 mi.; in 20°, 65 mi.; in 30°, 59.9 mi.; in 40°, 53 mi.; in 50°, 44.46 mi.; in 60°, 34$\frac{7}{12}$ mi.; in 70°, 23.66 mi.; in 80°, 12 mi.; in 90°, 0.]

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[1] In consequence of the flattening of the earth at the poles, degrees of *latitude* are not absolutely equal. A degree of latitude at any place is $\frac{1}{360}$ part of the circumference of a circle which has the same curvature as the meridian at that place.

270. ***Table of Longitude and Time.***

15° longitude make 1 hr. diff. in time.
1° longitude makes 4 min. diff. in time.
1′ longitude makes 4 sec. diff. in time.

271. Rule. — I. *To change longitude to time, multiply the degrees, minutes, and seconds by 4, calling the product minutes, seconds, and 60ths of a second.*

II. *To change time to longitude, express the time in minutes and seconds, and, dividing by 4, call the quotient degrees and minutes.*

The reason for this is apparent; since 1° corresponds to 4 min. of time, 1′ to 4 sec. of time, and 1″ is $\frac{1}{60}$ of 1′.

1. Adrian, Mich., is in 84° west longitude; and Fort Kearney, Neb., is in 99° west longitude. When it is 9 o'clock A.M. at Adrian, what time is it at Fort Kearney? When it is noon at Fort Kearney, what time is it at Adrian?

How many degrees *west* of Adrian is Fort Kearney?

2. New-York City is in longitude 74° west, and San Francisco in 122½° west. What is the difference in time? When it is 6 A.M. at New York, what time is it at San Francisco? When it is 4 P.M. at San Francisco, what time is it at New York?

3. In the latitude of Ann Arbor, Mich., 51.1 miles make a degree of longitude. Detroit is 38 mi. east of Ann Arbor. What is the difference in time?

4. What is the difference in longitude between two places whose difference in time is ¼ an hour?

5. What is the difference in longitude between two places whose difference in time is 3½ hr.? 2½ min.? 2 hr. 15 min. 20 sec.?

6. New York being in longitude 74° west, and New Or-

leans in 90° west, when it is 2 P.M. at New York, what time is it at New Orleans? When it is 11 A.M. at New Orleans, what time is it at New York?

7. What is the difference in time between Chicago, 87° 37′ 37″ W. long., and Cincinnati, 84° 27′ W. long.? When it is 5 P.M. at Chicago, what is the time at Cincinnati? When it is 8 A.M. at Cincinnati, what time is it at Chicago?

8. The difference in time between two places being 1 hr. 22 min. and 32 sec., what is the difference in longitude?

9. A telegraphic signal given from Washington, D.C., at 3.20 P.M., was received at St. Louis, Mo., at 2.27 P.M. The longitude of Washington being 77° 0′ 15″, what is that of St. Louis?

10. The difference in time between Buffalo and Cleveland being 11 min., what is the difference in longitude?

11. Boston, Mass., and Ann Arbor, Mich., are in about the same latitude; and the difference in time is 51 min. At this latitude 51.1 mi. make a degree of longitude. What is the distance in a direct line from Ann Arbor to Boston?

12. Berlin, Prussia, is in longitude 13° 23′ 53″ E., and Boston is in longitude 71° 4′ 9″ W. When it is 4 A.M. at Berlin, what time is it at Boston?

13. Annapolis, Md., and Cincinnati, O., are both in latitude 39° N. (nearly). On this parallel a degree of longitude is 53.47 mi., and the distance between the places is 422.413 mi. What is the difference in time?

14. The difference in time between Logansport, Ind., and Omaha, Neb., is 39 min.; and the distance between them is 509 mi. How many miles make a degree of longitude at this latitude?

15. In the latitude of Milwaukee, Wis. (43° 2′), 50½ mi. make a degree of longitude. How far east or west of Milwaukee do you have to go to make a difference in time of 15 min.? Of 20 min.? Of ½ an hour? Prairie du Chien has

the same latitude. A man's watch being set to Milwaukee time, he found it 13½ min. fast at Prairie du Chien. How far is it in a direct line from one place to the other?

16. The difference in time between Detroit and Chicago is 19 min. If my watch is set to Chicago time, how will it compare with Detroit time? The longitude of Detroit is 82° 58′ west. What is the longitude of Chicago?

17. Allowing no time for the passage of the current, at what time would a telegraphic signal sent from Ann Arbor, 83° 50′ 48″.3 W., at 3:45½ P.M., reach Cambridge, Mass., 71° 7′ 24″.9 W.?

18. Pekin, China, is 116° 27′ east longitude, and Washington is 77° west longitude. When it is noon on Jan. 1 at Washington, what time is it at Pekin?

19. Suppose the explosion of a meteor to have been observed at Ann Arbor at 2 o'clock 35 min. 27 sec. A.M., and at another point in the same parallel at 3 o'clock 4 min. 10 sec. A.M.: how many miles are the places apart? Is the second east, or west, of Ann Arbor? What is its longitude? See Exs. 17 and 11.

20. A and B sailed together from San Francisco. A kept his watch by San Francisco time, and B set his by the sun every day. After 10 days, A's watch was 4 hours 39 minutes faster than B's. In what longitude were they then, the longitude of San Francisco being 122° 26′ 15″ west?

21. At which point does the sun rise first, — at Philadelphia, 75° 9′ 23″.4 W., or St. Louis, 90° 15′ 10″ W.? How much?

SECTION VII.

PRACTICAL EXPEDIENTS.

[The purpose of this section is to exhibit a few of the practical expedients by which common arithmetical operations may be abridged. While the pupil should perform the operation intelligently, and hence should *occasionally* be asked to give a "solution," the chief purpose is to secure facility in getting results.]

CONTRACTIONS IN MULTIPLICATION.

272. *To Multiply by 25.* — Multiply by 100, and then divide by 4.

273. *To Multiply by 125.* — Multiply by 100, and add to this product $\frac{1}{4}$ itself.

Ex. Multiply 2346 by 25. 780 by 25. 800 by 25. 5432 by 25, 5437, 1000.

OPERATIONS.

234600
58650

Ex. Multiply 5082 by 125; also 3702, 540, 100, 64827, 34000.

508200
127050
635250

274. *To Multiply by a Number represented by two Digits, one of which is 1.* — Multiply by the digit which is not 1, and write the product under the multiplicand; removing this product one place to the left if this digit is 10's, and one to the right if it is units, adding these numbers.

Ex. Multiply 78579 by 81. By 61, 91, 71, 21, 31, 11, 41.

78579
628632
6364899

Ex. Multiply 78579 by 18. By 13, 15, 17, 16, 14, 15.

78579
628632
1414422

Ex. Multiply 30478654 by 41, 19, 81, 17, 91, 16, 61, 31, 13.

275. *To Multiply by a Number represented by 9's.* — Multiply by 1 with as many 0's at the right as there are 9's, and then subtract the multiplicand. If the right-hand figure of the multiplier is 8, subtract twice the multiplicand; if 7, three times; if 5, *five* times, etc.

Ex. Multiply 857639 by 999. Also 786042 by 9999; 3760048 by 99; 2856437 by 9999; 987654 by 999; 9999 by 9999.

857639000
857639
856781361

Ex. Multiply 857639 by 998. Also 64738 by 998; 64820 by 9998; 43725 by 98.

857639000
1715278
855923722

Ex. Multiply 8764987 by 997. By 99. By 95. By 98. By 995.

276. *To Square*[1] *any Number ending in* $\frac{1}{2}$. — Multiply the integral part by 1 more than itself, and to the product add (annex) $\frac{1}{4}$.

Ex. Square $34\frac{1}{2}$. $46\frac{1}{2}$. $87\frac{1}{2}$. $93\frac{1}{2}$. $16\frac{1}{2}$.

Ex. Square $17\frac{1}{2}$. $128\frac{1}{2}$. $478\frac{1}{2}$. $647\frac{1}{2}$. $273\frac{1}{2}$.

Ex. Square (mentally) $5\frac{1}{2}$. $4\frac{1}{2}$. $1\frac{1}{2}$. $2\frac{1}{2}$. $3\frac{1}{2}$. $7\frac{1}{2}$. $8\frac{1}{2}$. $9\frac{1}{2}$. $10\frac{1}{2}$.

34
35
170
102
1190¼

The reason for this rule will appear if we consider, that, when we multiply (in the ordinary way) by the $\frac{1}{2}$ in the multiplier, we get $\frac{1}{2}$ of the integer in the multiplicand $+ \frac{1}{4}$. Then, when we multiply the $\frac{1}{2}$ in the multiplicand by the integer in the multiplier, we get $\frac{1}{2}$ the integer. Hence the entire effect of the $\frac{1}{2}$ in each factor is to give a product larger than the integer by *once* the integer and $\frac{1}{4}$ more.

8½
8½
4½
4
64
72¼

CONTRACTIONS IN DIVISION.

[The reasons for the following are so simple, that it is not deemed necessary to state them. We give the rules, and a few examples for practice.]

[1] To square a number is to multiply it by itself. Thus the square of 32 is $32 \times 32 = 1024$.

277. *To divide by* 25. — Multiply by 4, and divide by 100; i.e., remove the decimal point 2 places to the left.

278. *To divide by* 15, 35, 45, 55, *or* 65. — Double the dividend, and divide by 30, 70, 90, 110, or 130.

279. *To divide by* 125. — Multiply by 8, and divide by 1000.

280. *To divide by* $3\frac{1}{3}$. — Multiply by 3, and divide by 10.

281. *To divide by* $12\frac{1}{2}$ *or* $16\frac{2}{3}$. — Multiply by 8 or by 6, and divide by 100.

Ex. 1. — Divide 7856 by 25. By 15, 35, 45, 55, 65, 125, $3\frac{1}{3}$, $12\frac{1}{2}$, $16\frac{2}{3}$.

2. Divide 8463, 963, 78427, 80056, and 7000, by each of the above numbers.

Aliquot Parts.

282. An **Aliquot Part** of a number is any number (integral or mixed) which will exactly divide it.

[The above methods are really methods by "*Aliquot Parts*;" but it is not customary to speak of them as such.]

283. The Aliquot Parts of \$1 are, —

$50¢ = \frac{1}{2}$.	$33\frac{1}{3}¢ = \frac{1}{3}$.
$25¢ = \frac{1}{4}$.	$16\frac{2}{3}¢ = \frac{1}{6}$.
$20¢ = \frac{1}{5}$.	$12\frac{1}{2}¢ = \frac{1}{8}$.
$10¢ = \frac{1}{10}$.	$6\frac{1}{4}¢ = \frac{1}{16}$.

Mental Exercises.

1. What cost 27 yd. of cloth at 25¢ per yard?

PRACTICAL OPERATION.[1] $\frac{1}{4}$ of 27 is $6\frac{3}{4}$. ∴ \$6.75.

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[1] SOLUTION. — At \$1 per yard, 27 yd. would cost \$27: hence, at \$$\frac{1}{4}$ per yard, 27 yards cost $\frac{1}{4}$ of \$27, or \$$6\frac{3}{4}$ = \$6.75. (See note at the beginning of the section.)

2. What cost 42 yd. of calico at $12\frac{1}{2}¢$ per yard? At $16\frac{2}{3}$? At $6\frac{1}{4}$? At $33\frac{1}{3}$? 25? 20? 10? 50?

OPERATION. — For the first $42 \div 8 = 5\frac{1}{4}$. ∴ The cost is $5.25.

3. What cost 35 lb. of butter at $33\frac{1}{3}¢$ per pound? At $25¢$? At $50¢$? At $20¢$? $16\frac{2}{3}$? 10? $12\frac{1}{2}$?

4. What cost 15 lb. of tea at $\$1.33\frac{1}{3}$ per pound? At $1.10? At $1.50? At $1.25? At $\$1.12\frac{1}{2}$? At $1.20?

OPERATION. — For the first, $15 + 5 = 20$. ∴ The tea cost $20.

5. At $12\frac{1}{2}¢$ per pound, how much sugar can bought for $1? For $3? For $1.25?

OPERATION. $8 + 2 = 10$. ∴ 10 lb. can be bought for $1.25.

6. At $33\frac{1}{3}¢$ per pound, how much coffee can be bought for $1? For $2? For $5?

OPERATION. $2 \times 3 = 6$. ∴ 6 lb. can be bought for $2.

7. What cost 15 yd. of cloth at $3.20 per yard? At $2.10? At $\$4.33\frac{1}{3}$?

8. What cost 25 yd. of cloth at $4.25 per yard? At $3.40? At $11.25?

OPERATION. — For the first, $425 \div 4 = 106.25$.

EXPLANATION. 100 yd. would cost $425, and 25 yd. would cost $\frac{1}{4}$ as much.

9. What cost $3\frac{1}{3}$ lb. of butter at $32¢$ per pound? At $36¢$? At $45¢$? At $21¢$?

At $21¢$ 10 lb. cost $2.10, and $3\frac{1}{3}$ cost $2.10 \div 3 = 70¢$.

10. What cost $12\frac{1}{2}$ yd. of cloth at $40¢$ per yard? At $1.12? At $16¢$? At $2.20?

11. What cost 48 gal. of molasses at $66\frac{2}{3}¢$ per gallon?

OPERATION. $48 - 16 = 32$. ∴ $32. Why?

12. What cost 120 bu. of potatoes at $37\frac{1}{2}¢$ per bushel?

13. What cost 256 bu. of onions at $87\frac{1}{2}¢$ per bushel?

$\$256 - \frac{1}{8}$ of $\$256$, or $\$256 - \$32 = \$224$. Why?

14. What cost 75 cords of wood at $5.50 per cord.

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284. The Aliquot Parts of a Year are, —

6 mo. $= \frac{1}{2}$; 4 mo. $= \frac{1}{3}$; 3 mo. $= \frac{1}{4}$; 2 mo. $= \frac{1}{6}$; 1 mo $= \frac{1}{12}$.

Also, when 30 da. are called a month, 1 mo. 6 da. $= \frac{1}{10}$; 1 mo. 10 da. $= \frac{1}{3}$; 1 mo. 15 da. $= \frac{1}{2}$; 2 mo. 12 da. $= \frac{1}{5}$.

285. The Aliquot Parts of a Month (30 da.) are, —

15 da. $= \frac{1}{2}$; 10 da. $= \frac{1}{3}$; 6 da. $= \frac{1}{5}$; 5 da. $= \frac{1}{6}$; 3 da. $= \frac{1}{10}$.

Mental Exercises.

Ex. 1. A man's wages are $450 per year. What are they for 6 mo.? 4 mo.? 3 mo.? 2 mo.? 1 mo.? 1 mo. 6 da.? 1 mo. 10 da.? 1 mo. 15 da.? 2 mo. 12 da.?

2. The interest on a note is $84 per year. What is it for each of the times in Ex. 1?

3. The interest on a note is $45 per year. What is it per month? What for 15 da.? 10 da.? 6 da.? 5 da.?

Written Exercises.

4. At $360 per year, what is the rent of a house for 2 yr. 7 mo. 25 da?

The work in the margin is all that should be written. The pupil *thinks* thus: "For 2 yr. the rent is $720; for 6 mo., $180; for 1 mo., $30 (i.e., $\frac{1}{6}$ of $180); for 15 da., $15; for 10 da., $10."

OPERATION.

$720
180
30
15
10
$955

5. At $450 per year, what is the rent of a house for 2 yr. 8 mo. 25 da.? For 1 yr. 5 mo. 11 da.? For 3 yr. 2 mo. 13 da.?

What part of the rent for 2 mo. is the rent for 10 da.? *What for 3 da.?*

OPERATION FOR LAST.

$1350
75
12.50
3.75
$1441.25

6. At $480 per year, what is the rent of a house for 8 mo. 13 da.?

7. If John's salary is $560 per year, how much does he receive for 5 mo. 15 da.? For 1 yr. 10 mo. 18 da.? For 2 yr. 6 mo. 10 da.? For 9 mo. 14 da.?

SOLUTIONS OF THE LAST.

BY ALIQUOT PARTS.

$46.666 = $280 − $140 − 15.555 − 3.111 − 3.111 = $441.777

14 da. = ⅓ mo. + ⅔ of ⅓ mo.

BY CANCELLATION.

$\frac{\$560 \times 9}{12} = \$420.00$

$\frac{\$560 \times 14}{12 \times 30} = \frac{196}{9} = \$21.78$

$\$420.00 + \$21.78 = \$441.78$

BY DECIMALS.

30|14
9.4⅔
$46.666+
9.4⅔
15555
15555
186666
419999
$441.78

Observe in the method by decimals, that, in multiplying a repetend, we write the first figure of each partial product as it would be if the repetend were indefinitely extended.

286. What number of ounces are aliquot parts of an avoirdupois pound?

What number of pecks are aliquot parts of a bushel?

8. What cost 6 lb. 12 oz. of butter at 28¢ per pound? What 5 lb. 14 oz. at 30¢? 4 lb. 10 oz. at 24¢? 7 lb. 13 oz. at 40¢? Same at 32¢ per pound?

$$\$2.80 + .20 + .10 + .02\frac{1}{2} = \$3.12\frac{1}{2}.$$

9. At 14¢ per pound, what is the cost of a dressed turkey weighing 12 lb. 14 oz.? What one weighing 8 lb. 11 oz.? Same at 16¢ per pound?

10. What cost 5 bu. 3 pk. 2 qt. of grain at $1.20 per bushel? What 2 bu. 2 pk. 3 qt.?

$$\$6.00 + .60 + .30 + .07\frac{1}{2} = \$6.97\frac{1}{2}. \quad \text{Why?}$$

11. What cost 15 yd. of cloth at $3.20 per yard? At $2.10? At $4.33⅓? $3.12½? $1.66⅔?

Solve the following by cancellation (see 2d solution, Ex. 7) : —

12. $56 per year is how much for 8 mo. 24 da.? 10 mo. 18 da.? 9 mo. 20 da.? 25 da.?

13. $320 per year is how much for 2 yr. 8 mo. 18 da.? 1 yr. 10 mo. 21 da.? 5 yr. 9 mo. 15 da.?

14. $20 per year is how much for 5 mo. 13 da.?

15. $150 per year is how much for 2 yr. 11 mo. 13 da.?

SOLUTION OF 14TH.

$$\frac{\overset{5}{\cancel{\$20}} \times 5}{\cancel{12}} = 8\tfrac{1}{3} \qquad = \$8.33$$

$$\frac{\overset{2}{\cancel{20}} \times 13}{\underset{6}{\cancel{12}} \times \underset{3}{\cancel{30}}} = \frac{4.333+}{6} = \frac{.72}{\$9.05}$$

SOLUTION OF 15TH.

$$\frac{\overset{5}{\cancel{\$150}} \times 13}{12 \times \underset{2}{\cancel{30}}} = \frac{65}{12} = \$\ 5\tfrac{5}{12}$$

$$\frac{\overset{25}{\cancel{150}} \times 11}{\underset{2}{\cancel{12}}} = \frac{275}{2} = \$137\tfrac{6}{12}$$

$$\$442.92$$

16. At $160 per year, what is the interest for 2 yr. 7 mo. 17 da.?

$$\frac{\overset{4}{\cancel{\$160}} \times 17}{\underset{3}{\cancel{12}} \times \cancel{30}} = \frac{68}{9} = \$7\tfrac{5}{9}, \text{ for 17 da.}$$

$$\frac{\overset{40}{\cancel{160}} \times 7}{\underset{3}{\cancel{12}}} = \frac{280}{3} = \$93\tfrac{1}{3}, \text{ for 7 mo.}$$

$\$160 \times 2 = \$320$, for 2 yr.

$\$420.89$, for 2 yr. 7 mo. 17 da.

17. Solve as above the following: —

$80 per year for 3 yr. 11 mo. 18 da.

$70 per year for 2 yr. 10 mo. 16 da.

$130 per year for 1 yr. 8 mo. 21 da.

$84 per year for 5 yr. 9 mo. 20 da.

$20 per year for 11 mo. 15 da. For 7 mo. 21 da.

$200 per year for 1 yr. 4 mo. 23 da.

SECTION VIII.

THE METRIC[1] OR DECIMAL SYSTEM OF WEIGHTS AND MEASURES.

DIAMETER, 2 CENTIMS.  WEIGHT, 5 GRAMS.

287. The metric system can be readily learned, if the student will first fix in mind a definite conception of

The Units.

The **Meter** (*mee'-ter*) is the unit of *Length*, and is the basis from which all the rest are deduced.

A **Meter** = 39.37 *inches*.

The **Liter** (*lee'-ter*) is the unit of *Measures* of *Capacity*.

A **Liter** = { 1.0567 *liquid quarts*, or
.908 *dry quarts*.

The **Gram** is the unit of weight.

A **Gram** = 15.432 *grains*.

[1] This system takes its name *metric* from the *meter*, the unit of linear measure established by the French Government, and made the basis of all the others. — See *Appendix*.

Subdivisions and Multiples of the Units.

288. These units are divided and subdivided into 10ths, 100ths, and 1000ths, and multiplied by 10, 100, 1000, and 10,000, to make the other denominations. Hence the system is a *Decimal System.*

The names of the denominations *lower* than the unit are formed by prefixing the Latin syllables *Deci* ($\frac{1}{10}$), *Centi* ($\frac{1}{100}$), and *Milli* ($\frac{1}{1000}$), to the name of the unit.

The names of the denominations *higher* than the unit are formed in like manner by prefixing the Greek syllables *Deka* (dek'a) (10), *Hekto* (100), *Kilo* (1000), *Myria* (10,000), to the name of the unit.

289. COMPLETE TABLE OF THE METRIC SYSTEM.

Relative Values.	Length.	Weight.	Capacity.	Surface.	Solidity.
10,000	Myriam (Mm)	..	..	..	..
1,000	Kilōm (Km)	Kilōg (Kg)	Kilōl (Kl)	..	..
100	Hektōm (Hm)	Hektōg (Hg)	Hektōl (Hl)	Hektar (Ha)	..
10	Dekam (Dm)	Dekag (Dg)	Dekal (Dl)	..	Decaster (Ds)
Unit.	*Meter* (m)	*Gram* (g)	*Liter* (l)	*Ar* (a) [1]	*Ster* (s) [1]
.1	Decim (dm)	Decig (dg)	Decil (dl)	..	Decister (ds)
.01	Centim (cm)	Centig (cg)	Centil (cl)	Centar (ca)	..
.001	Millim (mm)	Millig (mg)	Millil (ml)	..	..

This table contains all the denominations in use, with the spelling and abbreviations approved by the Metric Bureau, Boston, and the American Metrological Society, New York. The abbreviated names will be seen to consist of the prefixes with the first letter of the principal word, or name of the unit. Thus we have *decim* for decimeter, *kilōg* for kilogram, *centil* for centiliter, etc. The *accent* is always on the first syllable: *c* is soft (*s*), *e* in the prefixes short, and *o* long (ō). Thus *decim* is dĕcĭm; *centig* is sĕntĭg; *kilol*, kĭlōl, etc. Ar is like *are.*

[1] 1 ar = 1 square dekam; 1 ster = 1 cu. meter.

290. The abbreviations of the names of the units and the multiples are written as superiors. The abbreviations of the multiples are written with capitals. The submultiples are written without capitals, and in line with the figures. Thus 5 meters, written 5^{m}; 3 liters, 3^{l}; 7 hektols, 7^{Hk}; 8 dekags, 8^{Dk}. So 4 decims is written 4 dm; 6 centigs, 6 cm; 9 millims, 9 ml, etc. The tendency now is to write all the abbreviations without the period.

LEGALIZED EQUIVALENTS.

291. For purposes of reduction to and from our common measures, the pupil should fix in mind the following equivalents as legalized by the United-States Government: —

$1^{m} = 39.37$ in.
$1^{l} = 1.06$ liquid qt.
or .908 dry qt.
$1^{g} = 15.432$ gr., or .035 oz. Av.

$1^{Kg} = 2.2$ lb. Av.
$1^{a} = 3.95$ sq. rd.
$1^{s} = 35.52$ cu. ft.

MEASURES OF LENGTH.

292. THE INSTRUMENTS USED in place of our common 2 ft. ruler or carpenter's square, and the yard-stick or measure, are the *Meter*, a ruler 39.37 in. long, and divided into 10ths (decims) and 100ths (centims); and a *Double-decim* ruler, 2 dm in length, graduated into centims, and these again divided into tenths, making millims. The meter is folded into 4 parts, or 10 parts, for a pocket-measure; and the 2 dm ruler into 2 parts.

Such lengths as we usually indicate by yards, feet, and inches, are indicated by meters and centims.[1] Such as we indicate by miles are indicated by kilōms (kilometers). Very small dimensions, as those used in microscopy, are indicated in millims.

1. To which of our common measures is a meter nearest equal? How many meters in a rod? What is the length of

[1] There seems to be a well-defined tendency to use the decim as a common unit for smaller measures. The use of the *double-decim* ruler, and the near commensurability of the decim with one foot, will facilitate this.

a 12 ft. board expressed in meters? Express the dimensions of a room 20 ft. by 24 ft. in meters.

2. What is the stature of a 6 ft. man expressed in the metric system? What of one 1.8^{m} expressed in feet and inches?

Is a man's stature any more likely to be conveniently expressed in feet and inches than in meters and decimals? Is a man any more likely to be just 6 ft. in height than 1.8^{m}?

3. How does the ten-folded pocket-meter compare in length (when folded) with the common four-folded 2 ft. pocket-ruler? How does the four-folded meter compare with the two-folded 2 ft. pocket-ruler?

4. When the metric system comes into common use, what lengths of boards will probably take the place of our 12 ft.? 14 ft.? 16 ft.? What thicknesses will probably take the place of our 1 in., 2 in., and 3 in. stuffs, respectively? What dimensions of scantling our 2 in. by 3 in.? Our 3×4? Our 2×8 joist? Our 2×12?

As an inch is very nearly $\frac{1}{4}$ of a decim, we shall probably speak of "quarter-decim" stuff, or simply "quarter-stuff," instead of "inch-stuff," "half-decim" stuff, or "half-stuff," for "2 in. stuff," etc. Of course, positive answers cannot be given to such questions as the above. Nevertheless, the student will get a better appreciation of the relation of the metric to the common system by exercising his judgment on such questions than by any mere reductions.

5. With what in our common measure will a 2 dm by 3 dm timber most nearly correspond? What a 3 dm by 4 dm? What a 3 dm square?

6. What will 12 in. by 16 in. glass be in the metric system? — that is, what size will be likely to replace this? What 18 in. by 24 in.?

7. What is the distance from Albany to New York, expressed metrically, it being 145 miles? What the distance from Detroit to Chicago *viâ* the M. C. R.R. (284 mi.)?

8. What simple fraction of a mile is a kilōm (approximately)?

9. A railroad-train, running 40^{Km} per hour, runs how many miles?

10. How will the rate 1 mile in 2 min. 4 sec. be expressed metrically? One mile in 5 min.?

11. Which is the faster rate, 1^{Km} in 2 min. 20 sec., or 1 mi. in 2 min. 40 sec.?

12. What part of an inch is a millim? What is the approximate value in hundredths of an inch?

13. Glass is ruled for microscopic measurements in parallel lines from $\frac{1}{100}$ to $\frac{1}{10000}$ mm apart. What are these distances in inches?

14. Animal cells vary in diameter from $\frac{1}{40}$ mm to $\frac{1}{80}$ mm, and vegetable from $\frac{5}{6}$ mm to $\frac{1}{200}$ mm. Express these facts in inches, calling a millim .04 of an inch.

MEASURES OF WEIGHT.[1]

293. For the ordinary purposes of the grocery and market, the *kilōg* (called kilō in Europe) is used. For jewellers' and apothecaries' purposes, and for the chemical laboratory, the *gram* and *millig* are the units used.

The standard government weights at Washington are of brass and platinum. The brass weights are a five-kilōg, double-kilōg, kilōg, demi-kilōg, double-hektōg, hektōg, demi-hektōg, double-dekag, dekag, demi-dekag, double-gram, and gram. The platinum are a demi-gram, double-decig, decig, demi-decig, double-centig, centig, demi-centig, double-millig, and millig.

1. When steak is 14¢ per lb., what is it per Kg? Sugar at 30¢ per Kg is what per lb.? At 25¢ per Kg?

2. What is the weight of a bushel of wheat in Kgs? Of oats? Of corn? Of a barrel of flour?

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[1] The measures of *length* and *weight* are the two of the metric system of the most practical importance in our country at present: hence this arrangement, and *the fuller attention* given them.

3. One lb. av. equals how many kilōgs? One kilōg is how many pounds?

4. How many grams in an ounce of gold? In a pennyweight? How many milligs make a grain troy or apothecaries'?

5. The United-States post-office allows 15^{g} as the weight of a single letter, or $\frac{1}{2}$ oz. troy. Which is the greater?

6. What would be the *practical* equivalent for the apothecaries' grain, scruple, and dram, in metric weights?

7. How many grams in an ounce avoirdupois? What would be the practical equivalent in grams for $\frac{1}{4}$ lb. avoirdupois?

8. One-eighth of a grain is the common dose of morphine. What would be the prescription in the metric system?

9. Quinine is frequently given in 4 or 5 grain doses. What would be the prescription in the metric system?

10. In weighing a quantity of sugar, I find it is balanced by a double-kilōg, a demi-hektōg, and a dekag weights. What is the weight in kilōgs?

N.B. — It is one purpose of a number of the preceding exercises to suggest, that, when the metric system comes into use, most of our common specifications of quantities for practical purposes will undergo slight changes to conform to the units of the new system, so as not to involve troublesome fractions. Thus, instead of a *rod*, we shall speak of 5 meters; and, instead of laying out village-lots 4 rods by 8, we shall lay them out 20^{m} by 40. Instead of 12 by 16 in. glass, we shall have 3 by 4 dm glass. Instead of prescribing 3 ij, the physician will write 8^{g}, etc.

11. Instead of 1 3, what amount will probably be substituted in the metric system?

294. The *Tonneau*, or *ton*, of the metric system, is $1000^{Kg} = 2{,}204.6$ lbs., and is, consequently, so nearly equivalent to our *long ton* (2,240 lbs.) as to take its place without difficulty. The name *Ton* will doubtless be used instead of the French *tonneau*. Since $1^{Kg} = 1^{l}$, or 1 *cu. dm.* of water, 1000^{Kg}, or $1^{T} = 1$ cu. meter of water.

12. Coal at $9 per common ton (2,000 lbs.) would be how much per metric ton? Hay at $12 per common ton would be how much per metric ton?

MEASURES OF CAPACITY.

295. For such quantities of liquids or dry substances as we usually designate by the pint, quart, or gallon, the *liter* is used, as 7.5^l, 15^l, $\frac{1}{2}^l$, etc.; but for larger quantities the *hektol* is used, as 8.2 hl, 10.5 hl, etc.

The United-States Government standards at Washington are a double-liter, liter, demi-liter, double-decil, decil, demi-decil, double-centil, and centil.

The double-liter, liter, and demi-liter are respectively so nearly equivalent to our ½ gal., quart, and pint measures for liquids, as to take their places without embarrassment.

A double-dekal and a dekal would take the place of our ½ bu. and peck measures for grain very readily.

1. To what common measure is a liter nearly equal? How many liters in a gallon? In a barrel of 31½ gallons? How many does a common pail (2½ gal.) hold?

2. How many liters in a peck? In a half-bushel?

3. If a bushel of wheat is to weigh 60 lb., what should be the weight of a hektōl?

4. Wheat at $3.50 per Hl is what per bu.?

5. Molasses at $1.25 per gal. is how much per Dl?

296. One of the principal advantages which the metric system offers for scientific purposes is the facility which it affords for passing from measures of capacity to those of weight, and *vicè versa*. Thus a *liter* is a *cubic decim*, and a *gram* is a *cubic centim* of pure water at the temperature of melting ice. Hence, knowing the specific gravity of any substance (i.e., its weight as compared with water), we can readily pass from weight to volume, and *vicè versa*.

6. What is the weight of a liter of distilled water at the temperature of melting ice?

7. The specific gravity of linseed-oil is .94. How much would a cask of 2^{Hl} weigh?

8. I find that a liter of alcohol weighs 8^{Hg}. What is its specific gravity?

The weight of a liter of any liquid expressed in kilōgs is its specific gravity, or the weight of a millil expressed in grams, etc.

9. The specific gravity of milk is 1.032. What does 1^{Dl} weigh?

10. A pail containing 1^{Dl} of cider is filled, and the cider found to weigh 10.18^{Kg}. What is the specific gravity of cider?

11. One millil of sulphuric acid is found to weigh 1.842^{g}. What is its specific gravity?

MEASURES OF AREA.

297. For measuring *Boards* and small areas the square meter, decim, etc., are used; but for *Land* the *Ar*, which is a square dekam, and even the Hektar, which is 100 ars, is used. $1^{Ha} = 2.471$ acres, and the *Ar* is about $\frac{1}{40}$ of an acre. Since 1 mile is but a little over 16^{Hm}, a section of land is about 256^{Ha}. It is common, in surveying, to speak of 16^{Hm} as a mile.

1. How many hektars in a section of land?

2. Land at $250 a hektar is how much per acre?

3. How many square feet in a square meter?

4. Land at 8^{Nap} a hektar is how many dollars an acre?

5. How many hektars in a rectangular piece of ground 1000^{m} by 400?

6. How many square meters in 6 boards 4^{m} in length and 5 dm in width?

MEASURES OF VOLUME.

298. The *Ster*, which is a cubic meter, is the proposed unit of volume; but it has fallen into general disuse. — *President* BARNARD, *in Johnson's Cyclopædia.*

The chief interest, therefore, which attaches to the metric measurement of volume at present, is as a means of defining the measures of weight and capacity. See (**296**) and also Appendix.

1. How many hektols of water does a rectangular vat contain which is 2 meters square and 2.5 meters deep?
2. In 200 sters how many cords?
3. 500 cu. yd. are how many sters?
4. How many cubic meters (sters) of earth are removed in digging a ditch 2^{m} wide, 1.5 deep, and 4^{Km} long?

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299. *Examples in the Use of the Metric Measures.*

[By permission, the following (46) examples are taken from an excellent little manual by Supt. Henry E. Sawyer of the schools of Middletown, Conn., now Associate Principal of the Connecticut State Normal School at New Britain. This manual will be found specially helpful in showing how to teach the Metric System so as to make it interesting and practically useful.]

1. Add 23 cm, 47 cm, 9 cm, 38 cm, 74 cm.
2. Add 237^{m}, 53 cm, $17^{m}3$, $24^{m}07$.
3. Add $239^{m}47$, 98 cm, 19^{m}, $70^{m}07$.
4. From 97 cm take 39 cm.
5. From $7^{m}35$ take $4^{m}86$. From $9^{m}08$ take 57 cm.
6. The lengths of the principal buildings at the Centennial Exposition were as follows: Main Building, $573^{m}024$; Machinery Hall, $426^{m}72$; Horticultural Hall, $116^{m}738$; Art Gallery, $111^{m}25$; Women's Pavilion, $58^{m}52$; Agricultural Building, $249^{m}936$. How far would they have extended if placed in a row, end to end?

7. How much longer was the Main Building than Machinery Hall? The Agricultural Building than the Art Gallery?

8. Multiply 24 cm by 3. 87 mm by 9.

9. $29^{m}43 \times 7 = ?$ $216^{m}329 \times 8 = ?$

10. Divide 78 cm by 13. 168^{m} by 8.

11. $16^{m}8 \div 8 = ?$ $436^{m}324 \div 4 = ?$

12. $9864^{Km} \div 8 = ?$ $96^{Hm} \div 7 = ?$

13. If Mary lives 753^{m} from the schoolhouse, how far must she travel, in coming to school and going home, in one week?

14. How far must a man walk in a day to travel $237^{Km}6$ in 6 days?

15. From Quebec to the mouth of the Saguenay River is about $216^{Km}18$. At what rate must a steamer run to make the trip in 9 hours?

16. In what time will a train run from Boston to Albany at the rate of $34^{Km}3$ per hour, the distance being about $325^{Km}85$?

17. How many revolutions will an engine-wheel $4^{m}5$ in circumference make in running $416^{Km}7765$?

18. Read the following: $6^{l}3$; $19^{Hl}15$; $3^{cl}7$; $93^{Dl}47$.

19. In 27^{l} how many cl? Dl?

20. In 3954 cl how many Dl? Hl? l? dl?

21. In 697^{Hl} how many l? cl? dl?

22. At 3 cents a cl, what is the price per l?

23. At 25 cents per l, what cost 2 cl?

24. If a Dl of oil is worth \$1.63, what is it per l?

25. When corn is worth \$2.24 per Hl, what is it per l?

26. At \$1.35 per Dl, what is a liter of molasses worth?

27. If a chicken eats 2 dl of corn in a day, what will it cost to keep 173 chickens a week when corn is worth \$2.15 per Hl?

28. Read $31^{g}7$; 23^{Kg}; $17^{T}396$; 37^{g}; $3^{Kg}7$.

29. Reduce $27^{T}3$ to Kg; to g.

30. Into a basket weighing $4^{Kg}3$ were put the following articles: 2^{Kg} of coffee, $\frac{1}{2}^{Kg}$ of tea, $3^{Kg}5$ of sugar, 100^{g} of pepper, 50^{g} of nutmegs, 500^{g} of turnip-seed, and $1^{Kg}5$ of rice. What was the weight of the basket and its contents?

31. A car weighing $7^{T}832$ contains 136 barrels of flour, each weighing $96^{Kg}16$. Find the weight of the car and its contents. What is the difference in weight?

32. At \$1.15 per Kg, what cost $7^{T}3$?

33. What cost a Kg of sugar at \$265 per T?

34. What cost $29^{Kg}5$ at $16\not{c}$ per g?

35. What is the weight of 27^{l} of water?

36. What is the weight of 36 cl of water?

37. $3^{l}5$ of water weighs how many g?

38. $1^{l}93$ of water weighs how many g?

39. What is the weight of 173^{l} of water?

40. A man carting water from a river has two casks, one holding 136^{l}, and the other 125^{l}. When both casks are full, what weight of water has he on his cart?

41. If milk is 1.03 times as heavy as water, how much should 8 liters of milk weigh?

42. If you buy 42 liters of milk, and find that it weighs only $42^{Kg}75$, is it pure?

43. Wishing to find the capacity of a bottle, and having no measures at hand, I weighed it. Empty, it weighed 520^{g}; and filled with water, $1^{Kg}29$. How much did it hold?

44. What is the capacity of a conical glass which weighs when empty 540^{g}, and when filled with water $2^{Kg}225$?

45. A test-tube weighing 19^{g} was filled with water, when the tube and water together were found to weigh $64^{g}5$. How much water was in the tube?

46. The same tube, filled with sulphuric acid, weighed $103^{g}5$. The acid was how many times as heavy as water?

CHAPTER V.

SECTION I.

PERCENTAGE.

Definitions and First Principles.

1. During a severe winter a farmer lost 5 sheep out of every 100 of his flock. What part of his flock did he lose?

2. John's father agreed to give him $8 for every $100 he would earn for himself. To what part of his earnings was his father's gift equal?

300. Per Cent means *By the Hundred.*

To say that a man lost 5 per cent of his sheep is to say that he lost 5 out of every hundred of them, or .05 of them. Again: to say that a father gives his son a sum equal to 8 per cent of the son's earnings is to say that he gives the son $8 for every $100 he earns, or a sum equal to .08 of his earnings.

3. A nurseryman lost by drought 6 per cent of his trees. What part of his trees did he lose? If he had 2150 trees, how many did he lose? i.e., .06 of 2150 = how many?

4. A man's house was damaged by fire to an extent estimated at 15 per cent of its value. If the house was worth $6530, what was the amount of damage?

5. A speculator bought a car-load of wheat for $450, and sold it at a profit of 4 per cent. How much did he make by the speculation?

This means that he made .04 of $450, which is $18.

6. What is 9 per cent of 250? 10 per cent of 48? 17 per cent of 53? 11 per cent of 1437?

301. **Rate**[1] is the number by which we multiply to obtain any required per cent of a given number.

Thus, to obtain 7 per cent of 250, we multiply 250 by .07. Hence .07 is the *Rate* (not *Rate per cent*). See footnote.

302. The result obtained by taking a certain per cent of a number is called *the Percentage.* The term *Percentage* is also used as a general designation for all processes involving this method of reckoning by the hundred.

303. The **Base** is the number upon which the percentage is estimated. In the last illustration, 250 is the *base.*

7. Mr. Smith, having a flock of 340 sheep, found that in 1 year they increased at the rate of 50 in a hundred. What was the per cent (or rate per cent) of increase? What was the *rate* of increase? How *much* was the increase? How many sheep had he after the increase?

304. The **Amount** is the sum of the base and percentage.

In Ex. 7 what is the Base? What the *Rate per cent?* What the *Rate?* What the *Percentage?* What the *Amount?*

8. Having $350, I used it in buying grain, which I sold so as to gain 5½%. How much had I then?

5½% of $350 = $19.25. Hence I had $350 + $19.25, or $369.25.
What is the *Base?* The *Rate?* The *Percentage?* The *Amount?*

[1] The use of the term *Rate per cent* in this sense is inadmissible; but we may so use *Rate:* in fact, this is the common meaning of the word *rate* in mathematics. An allowance of 7 on a hundred is not at a rate of .07 *per cent,* although it is at a *rate of .07: the Rate per cent* is 7. Per cent is used by ellipsis for *Rate per cent.*

9. If the *base* is 784, and the *rate per cent* 6, what is the *percentage? What the amount?*

305. The character % is used as a substitute for the words *per cent.*

Thus 4% means "4 per cent."

10. Mr. A. used $500 speculating in wheat, and lost 8% of it. How much did he lose? How much had he left?

306. The **Difference** is what remains of the *Base* after the percentage is taken out.

11. Mr. Smith had an orchard of 320 trees; and, during a severe winter, 5% of them died. How many died? How many remained? What is the *Base? Rate? Percentage? Difference?*

12. What part of a number is 60% of it?

60% means 60 on a hundred. Now, 1 on a hundred is $\frac{1}{100}$ part. Hence 60 on 100 is $\frac{60}{100}$, or $\frac{3}{5}$ part. Hence 60% of any thing is $\frac{3}{5}$ of it.

13–29. What part of a number is 10% of it? 20%? 50%? $12\frac{1}{2}$%? $16\frac{2}{3}$%? 40%? 11%? 15%? 7%? 6%? 4%? $\frac{1}{2}$%? $\frac{3}{4}$%? $\frac{5}{8}$%? 100%? 200%? 150%?

$\frac{1}{2}$%, or $\frac{1}{2}$ per cent, is $\frac{1}{2}$ on a hundred, just as 2% is 2 on a hundred. At $\frac{1}{2}$% the rate is .005.

30–40. What *rate* is $\frac{3}{5}$%? $\frac{3}{2}$%? $\frac{1}{5}$%? $1\frac{1}{2}$%? $\frac{3}{4}$%? $\frac{5}{8}$%? $\frac{4}{5}$%? $\frac{5}{6}$%? $1\frac{1}{4}$%? $12\frac{1}{2}$%? $8\frac{3}{4}$%?

$\frac{1}{4}$% is at the rate .00$\frac{1}{4}$, or .006. $\frac{3}{4}$% is at the rate .01$\frac{1}{2}$, or .015. The per cent being $\frac{1}{3}$, the rate is .00$\frac{1}{3}$. $\frac{3}{4}$% is at the rate .0075.

41–48. What is the *per cent* corresponding to each of the following rates: .05? .10? 1.12? .03? .007? .005? .00$\frac{1}{4}$? .$\frac{1}{2}$?

.$\frac{1}{2}$ is an absurd expression, and has no meaning. It is not $\frac{1}{2}$ of $\frac{1}{10}$, for that is written .0$\frac{1}{2}$; nor is it $\frac{1}{2}$ of 1, for that is simply $\frac{1}{2}$.

307. ***Rules of Percentage.***

I. $Base \times Rate = Percentage.$
II. $Base + Percentage = Amount.$
III. $Base - Percentage = Difference.$

} PRIMARY RULES.

IV. $\frac{Percentage}{Rate} = Base.$
V. $\frac{Percentage}{Base.} = Rate.$
VI. $\frac{Amount}{Base} = 1 + Rate.$
VII. $\frac{Amount}{1 + Rate} = Base.$
VIII. $\frac{Difference}{Base} = 1 - Rate.$
IX. $\frac{Difference}{1 - Rate} = Base.$

} SECONDARY RULES.

The *Three Primary Rules* are but direct applications of definitions (**301**), (**304**), and (**306**).

IV. and V. are deduced from I. on the principle that the product of two factors divided by either factor gives the other. Thus, —

$$Base \times Rate = Percentage. \quad \therefore \begin{cases} \frac{Percentage}{Rate} = Base. \\ \frac{Percentage}{Base} = Rate. \end{cases}$$

VI. and VII. are obtained from II. on the same principle that IV. and V. are from I., by observing, that, to obtain the *Amount*, we multiply the *Base* by the *Rate*, and then add the *Base*, whence we obtain (1 + *the Rate*) times the *Base*. Thus, —

$$Base \times (1 + Rate) = Amount. \quad \therefore \begin{cases} \frac{Amount}{Base} = 1 + Rate. \\ \frac{Amount}{1 + Rate} = Base. \end{cases}$$

VIII. and IX. are obtained from III. as VI. and VII. are from II., *by observing, that, to obtain the Difference*, we take the *Rate* times the

Base out of the *Base*, and hence have (1 — *the Rate*) times the *Base*. Thus, —

$$Base \times (1 - Rate) = Difference. \quad \therefore \quad \begin{cases} \frac{Difference}{Base.} = 1 - Rate. \\ \frac{Difference}{1 - Rate} = Base. \end{cases}$$

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308. *Methods of Explanation.*

[There are two principal methods of applying the principles of percentage in practice: one being to observe which of the elements — base, rate, percentage, amount, or difference — are given, and which required, and then select and apply the proper rule; while the other method teaches to examine into the nature of each problem, and, discovering the relations existing between the quantities, analyze accordingly. We give illustrations of both methods.]

1. How much is 7% of $350?

SOLUTION BY RULE. — In this case $350 is the *base*, and .07 is the rate, and the percentage is required. Hence, by (**307**), I., we multiply the base by the rate, and find the percentage to be $24.50.

OPERATION.

$350
.07
$24.50

SOLUTION BY ANALYSIS. — Since 7% is 7 on every hundred, it is .07 of any sum. .07 of $350 is $24.50.

2. A speculator bought a car-load of wheat for $450, and sold it at a profit of 4%. How much did he receive?

BY RULE. — In this case $450 is the base, .04 is the rate, and the amount is required. Hence we multiply the base by the rate, and have the percentage by (**307**), I. Adding the percentage to the base, we have the amount by (**307**), II.

OPERATION.

$450
.04
Percentage, $18.00
Base, 450
Amount, $468.00

BY ANALYSIS. — Since a profit of 4% is an increase of 4 on a hundred (i.e., of .04), the speculator gained .04 of $450 by the operation. .04 of $450 is $18. Hence he gained $18 on $450, and consequently received $450 + $18 = $468.

3. By investing a certain sum, I made a profit of 12%, and thereby gained $100.80. What was the sum invested?

By Rule. — Here we have the *percentage* ($100.80) and the *rate* (.12) given to find the *base*. Hence, according to (**307**), IV., we divide the percentage by the rate, and obtain the base, $840.

Operation.

$$\frac{\$100.80}{.12} = \$840.$$

Analysis. — A gain of 12% is a gain of 12 cents on $1. Hence to gain $100.80 will require as many dollars as $0.12 is contained times in $100.80, which is $840.

4. Bought cloth at $3.50 per yard, and sold it at 70¢ per yard more than I gave. What per cent profit did I make?

By Rule. — Here we have the base $3.50, and the percentage $0.70 to find the rate. Hence we divide the percentage (.70) by the base (3.50), and have the rate, .20, by (**307**), V. The rate being .20, the rate per cent is 20.

Operation.

$$\frac{.70}{3.50} = .20.$$

Analysis. — If, in spending $3.50, I gain $0.70, in spending $1 I should gain .70 ÷ 3.50, or .20 dollars, and in spending $100 I should gain $20. Hence I gain 20 on 100, or 20%.

5. If I buy land at $50 per acre, and sell it at $60, what per cent do I make?

By Rule. — In this case the $60 includes the base and percentage: hence it is the *amount*. Having given base and amount, we divide the amount by the base, and get 1.20, which, by (**307**), VI., is 1 + rate. ∴ The rate is .20, and the rate per cent 20.

Operation.

$$\frac{60}{50} = 1.20.$$

Analysis. — If I receive $60 for $50 laid out, for $1 I receive 60 ÷ 50, or $1.20. Hence I make 20¢ on $1, or 100¢; i.e., 20%.

6. By selling a certain piece of land for $560, I make 12% on the cost. What was the cost?

By Rule. — Given the amount (560) and rate (.12) to find the base, by (**307**), VII., we divide the amount by 1 + the rate (i.e., 1.12), and have the base, 500.

Operation.

$$\frac{560}{1.12} = 500.$$

ANALYSIS. — As I make 12%, I make 12¢ on $1. Hence, for every dollar the land cost me, I get $1.12. But in all I receive $560. Hence the land cost me as many dollars as $1.12 is contained times in $560; i.e., $500.

7. I sell a fine horse for $540, and thereby lose 25%. What did the horse cost me?

BY RULE. — Here are given the difference (540) and the rate (.25). Hence, by (307), IX.,

OPERATION.

$$\frac{540}{.75} = 720.$$

ANALYSIS. — As I lose 25%, I lose 25¢ on each dollar the horse cost me, and hence receive 75¢ for each $1 cost. But in all I receive $540. Therefore the horse cost me as many dollars as 75¢ is contained times in $540, or $720.

8. Mr. A. bought a house for $7500, and sold it for $6000. What per cent was his loss?

BY RULE. — There are given base ($7500), difference ($6000), to find rate. Hence, by (307), VIII., etc. Finally as .80 is 1 — rate, that is the remainder after subtracting the rate from 1, if we take this remainder from the minuend, 1, we shall have the subtrahend, .20, which is the rate. Hence the rate per cent was 20.

OPERATION.

$$\frac{6000}{7500} = \frac{4}{5} = .80$$

$$1 - .80 = .20$$

ANALYSIS. — He received $6000 for an outlay of $7500. Hence for an outlay of $1 he received 6000 ÷ 7500, or $0.80. Hence he lost 20¢ on $1, or 20%.

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309. *Examples for Practice.*

Solve the following with or without writing according to the simplicity of the problem.

N.B. — The rate should always be expressed in the most convenient form, not necessarily in the form of a *decimal fraction*. Thus, to find $33\frac{1}{3}\%$ of $360, we would not multiply by $.33\frac{1}{3}$, but by $\frac{1}{3}$; i.e., *divide* by 3. But, to obtain 7% of any number, it is most convenient to mul-

tiply by .07. Again: to find 25%, we would divide by 4, which is the same as multiplying by .25, or $\frac{1}{4}$. Always use as few figures as practicable in the "operation." Use cancellation whenever it will aid.

1. 5% of 780 = 39.
2. 12% of 475 yd. = 57 yd.
3. 10% of 860 trees = ?
4. 35% of 1840 = ?
5.[1] $33\frac{1}{3}$% of \$234.54 = ?
6. 45% of $18\frac{3}{4}$ = ?
7.[2] $\frac{3}{4}$% of \$348 = ?
8. $\frac{4}{5}$% of $1\frac{1}{16}$ lb. = ?
9. 7% of \$47 = ?
10. 100% of \$58 = ?
11. 1% of \$58 = ?
12.[3] 120% of \$150 = ?
13. 110% of \$60 = ?
14. $112\frac{1}{2}$% of \$16 = ?
15. 150% of \$365.20 = ?
16. 125% of \$7.50 = ?
17. 115% of \$11.37 = ?
18. 200% of \$247 = ?
19. 300% of \$75.50 = ?
20. $\frac{1}{2}$% of \$360 = ?
21. 50% of \$360 = ?
22. 1% of \$37 = ?
23. 100% of \$37 = ?
24. 7% of \$43.20 = ?
25. 9% of \$162.43 = ?
26. Find 6% of \$1.75.
27. Find 6% of \$350.
28. Find 7% of \$140.
29. Find 8% of \$1.
30. Find 9% of \$100.
31. Find $\frac{2}{3}$% of \$0.75.
32. Find $1\frac{1}{2}$% of $\frac{5}{8}$.
33. Find $\frac{5}{8}$% of \$1540.
34. Find $\frac{1}{2}$% of \$2500.
35. Find 7% of \$34.28.
36. Find 11% of \$15.17.
37. Find 6% of \$42.18.
38. Find $33\frac{1}{3}$% of 465 gal.
39.[4] Find $37\frac{1}{2}$% of 816 mi.
40. Find 35% of \$21.75.
41. Find 48% of 13.42.
42.[5] Find 7% of $\frac{3}{4}$. $\frac{5}{8}$.
43. 8% of 7. $4\frac{1}{2}$. 5.
44. $12\frac{1}{2}$% of 16. $3\frac{1}{5}$. 7.
45.[6] $66\frac{2}{3}$% of $1\frac{1}{2}$. $4\frac{1}{5}$. 6.
46. 7% of $\frac{1}{2}$. $\frac{2}{3}$. $\frac{4}{5}$.
47. 9% of $4\frac{3}{4}$. $5\frac{1}{6}$. 7.
48. 3% of $2\frac{2}{3}$. $1\frac{1}{2}$. $5\frac{1}{3}$.
49. 11% of $\frac{5}{6}$. $3\frac{1}{4}$. $2\frac{2}{7}$.
50. 111% of $\frac{5}{17}$. $\frac{11}{13}$. $3\frac{3}{7}$.

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[1] $33\frac{1}{3}$% is .33$\frac{1}{3}$, or $\frac{1}{3}$ of any thing.

[2] 1% of \$348 is \$3.48, and $\frac{3}{4}$% is $\frac{3}{4}$ of \$3.48.

[3] 120% is 1.20, or $1\frac{1}{5}$ times any thing. ∴ \$150 + \$30 = \$180.

[4] $37\frac{1}{2}$% is $\frac{3}{8}$ of any thing. ∴ $\frac{3}{8}$ of 816 = 3 × 102 = 306.

[5] 7% of $\frac{5}{6}$ is $\frac{7}{100}$ of $\frac{5}{6} = \frac{7}{120}$.

[6] $66\frac{2}{3}$% of $1\frac{1}{2}$ is $\frac{2}{3}$ of $\frac{3}{2} = 1$.

310. What per cent of—

51. \$60 is \$20?
52. 55 is 11?
53. 148 is $24\frac{2}{3}$?
54. $47\frac{1}{2}$ is $7\frac{1}{2}$?
55. 40 is 15?
56. $\frac{1}{2}$ is $\frac{1}{3}$?
57. $\frac{1}{3}$ is $\frac{1}{2}$?
58. $\frac{3}{4}$ is $\frac{2}{5}$?
59. 37 yd. is 37 yd.?
60. 37 yd. is .37 yd.?

What per cent is—

61. \$180 of \$360?
62. \$1.80 of \$360?
63. $\frac{1}{2}$ of $\frac{1}{3}$?
64. $\frac{3}{4}$ of $\frac{2}{5}$?
65. \$547.80 of \$365.20?
66. \$66 of \$60?
67. \$226.50 of \$75.50?
68. 8¢ of 5¢?
69. 4¢ of $12\frac{1}{2}$ ¢?
70. 1¢ of \$1? \$1 of 1¢?

Such questions are equivalent to asking what part one number is of another (**153**), with the added condition that the answer shall be given in hundredths. Thus 15 is $\frac{15}{100}$, or $\frac{3}{8}$ of 40, and $\frac{3}{8} = 37\frac{1}{2}$ hundredths.

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311. Of what number is—

71.[1] 385 $12\frac{1}{2}\%$?
72. 245 10%?
73. 125 15%?
74. 7.15 $33\frac{1}{3}\%$?
75. \$53.25 10%?
76. 27.5 bu. 8%?
77. 168 men 8%?
78. 231 oxen 7%?
79.[2] $\frac{2}{5}$ $53\frac{1}{3}\%$?
80. 15 $37\frac{1}{2}\%$?

Of what number is—

81. 146 lb. 8%?
82. 240 men $12\frac{1}{2}\%$?
83.[3] $5\frac{1}{3}$ $1\frac{1}{2}\%$?
84.[4] $7\frac{3}{4}$ $\frac{3}{4}\%$?
85. $\frac{1}{2}$ 150%?
86. \$37 1%?
87. \$37 100%?
88. \$37 300%?
89. \$78.18 $33\frac{1}{3}\%$?
90. 5 8%?

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[1] Avoid mechanical processes. $12\frac{1}{2}\%$ is $\frac{1}{8}$. If 385 is $\frac{1}{8}$, 8-eighths is 8 times 385.

[2] $53\frac{1}{3}\%$ is $\frac{53\frac{1}{3}}{100}$, or $\frac{160}{300} = \frac{8}{15}$. $\frac{2}{5} \div \frac{8}{15} = \frac{2}{5} \times \frac{15}{8} = \frac{3}{4}$.

[3] OPERATION. $\frac{5\frac{1}{3}}{\frac{1\frac{1}{2}}{100}} = \frac{1600}{4\frac{1}{2}} = \frac{3200}{9} = 355\frac{5}{9}$.

[4] $\frac{7\frac{3}{4}}{\frac{\frac{3}{4}}{100}} = \frac{3100}{3} = 1033\frac{1}{3}$.

312. What sum amounts to —

91.[1] \$360 at 20%?
92.[2] \$210 at 75%?
93. \$61.53 at 5%?
94. \$36.54 at $1\frac{1}{2}$%?
95. \$40.23$\frac{1}{4}$ at $4\frac{1}{2}$%?
96. \$129.368 at 3%?
97.[3] \$3519.15 at $1.12\frac{1}{2}$%?
98. \$82 at 2.5%?
99. \$408.20$\frac{1}{2}$ at 7%?
100.[4] $\frac{3}{5}$ at $\frac{2}{3}$%?
101. \$540 at 8%?
102. \$234 at 6%?
103. \$1000 at 9%?
104. $4\frac{1}{2}$ at $2\frac{3}{4}$%?
105. \$100 at 100%?
106. \$100 at 1%?
107. \$56.85 at 10%?
108. \$145 at 4%?
109. $\frac{3}{4}$ at $12\frac{1}{2}$%?
110. \$4 at $112\frac{1}{2}$%?

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313. What sum gives a difference of —

111. \$483.60 at 7%?
112.[5] \$24.50 at $12\frac{1}{2}$%?
113. \$300 at $6\frac{1}{4}$%?
114. 730 yd. at 4%?
115. 584 lb. at 8%?
116. 347 ft. at 10%?
117. 46 at $3\frac{1}{2}$%?
118. 112 at $2\frac{3}{4}$%?
119.[6] $\frac{3}{4}$ at $37\frac{1}{2}$%?
120. $5\frac{1}{3}$ at $63\frac{2}{3}$%?
121. 273 at 25%?
122. 520 at 20%?
123.[7] 320 at $33\frac{1}{3}$%?
124. \$138 at 7%?
125. \$24.50 at 8%?
126. \$160 at 12%?
127. \$300 at 3%?
128. $\frac{3}{5}$ at $\frac{7}{8}$%?
129. $4\frac{3}{4}$ at $3\frac{1}{2}$%?
130. 0 at 100%?

314. General Suggestion. — In solving problems in percentage, a good general method is to *indicate all operations before performing any, and then put the work in the best form for cancellation.*

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[1] Keep the work down to a minimum. $360 \times \frac{5}{6} = 300$.

[2] $210 \times \frac{4}{7} = 120$.

[3] $\frac{3519.15}{1.01125} = 3480$.

[4] $\frac{\frac{3}{5}}{1.00\frac{2}{3}} = \frac{60}{100\frac{2}{3}} = \frac{30}{50\frac{1}{3}} = \frac{90}{151}$.

[5] $1 - .12\frac{1}{2} = 1 - \frac{1}{8} = \frac{7}{8}$. $\therefore \frac{24.50}{\frac{7}{8}} = \frac{196}{7} = 28$.

[6] $\frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \times \frac{8}{5} = 1\frac{1}{5}$.

[7] $\frac{320}{\frac{2}{3}} = 160 \times 3 = 480$.

SECTION II.

APPLICATIONS OF SIMPLE[1] PERCENTAGE.

315. Business calculations based on percentage are of two classes: (1) those which do not involve the element of *time* in the computations, and (2) those which do.

Of the 1st class are the simpler problems of *Profit* and *Loss*, *Commission*, *Brokerage*, *Bankruptcy*, *Stocks*, *Insurance*, *Taxes*, and *Duties*. Of the 2d class are *Interest*, *Discount*, *Annuities*, including many problems in *Insurance*, *Exchange*, and *Equation of Payments and Accounts*.

Problems of the 1st class are solved directly by the principles, or by the formulas (**307**).

Profit and Loss.

316. **Profit**, or **Gain**, is the excess of what is received for an article over its total cost. **Loss** is the excess of the total cost of an article over what is received for it.

1. Bought a flock of sheep numbering 350. In one season it increased 24%. How many had I then? Had it *decreased* 24%, how many should I have had?

2. Bought 12 crocks of butter, weighing 35 lb. each net,[2] for 19¢ per lb., and sold it at a profit of 2%. What did I receive for the whole? How much did I *gain*?

3. My house, which is valued at $5500, was damaged by fire 15%. What was the total damage?

4. Invested $3560 in town-lots in a new village in Kansas. In the course of a year they increased in value 62½%. What were they worth then?

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[1] By "Simple" Percentage is meant Percentage which does not involve the element of *time*.

[2] This means exclusive of the crocks which contain the butter.

5. Bought 20 horses at an average price of $225 each. I lost 25% of them, and sold the remainder at an advance of 30% on the cost price. Did I gain, or lose, by the transaction? How much?

6. What must be the selling price of cloth which cost $4 per yard, in order to realize a profit of 10%? Of 12½%? Of 25%? Of 30%? Of 15%? Of 20%?

7. At what must calico, which cost 6¢ per yard, be sold, in order to realize a profit of 8%? Of ⅝%? Of 1%? Of 2½%? Of ½%? Of 1¾%? Of 5%? Of 4½%?

8. Bought a piece of cloth containing 30 yd. at $3 per yard. 10 yd. of it were damaged, so that I had to sell it at a loss of 50%. The remainder I sold at 20% profit. How much did I lose on the whole?

9. The standard for gold coin in the United States is 9 parts pure gold, and 1 part alloy. What % is alloy? What % is gold?

10. If I buy cloth at $5 per yard, and sell it at $5.50, what % profit do I make?

11. When I sell goods at 1¼ their cost, what % profit do I make?

I make a profit of ¼ the cost; i.e., on every $1 spent in buying I make ¼ of a dollar. The question then is, "¼ is what % of 1?"

12. If I buy land at $27 per acre, and sell it at $36, what % do I make? What, if I sell it at $30? At $90?

13. A fruit-grower shipped 300 baskets (pecks) of peaches to Chicago; but, on the way, 75 baskets spoiled. What % did he lose? What % was left?

14. When I sell goods at ⅔ the cost, what % do I lose?

The loss is ⅓ of the cost. Then the question is, "⅓ is what % of 1?"

15. When I sell goods at twice the cost, what % do I make? When at 1⅕ the cost? At 1$\frac{1}{10}$ the cost? At 2½ the *cost?*

16. What % is made by buying tea at 80 cents per pound, and selling it at \$1? At 90¢? At 85¢? At \$1.10?

17. Bought a span of horses for \$575, and sold them at \$650. What % did I make?

18. Bought a house and lot for \$11500, and sold them at \$13640, after having expended \$350 in repairs. What % did I make?

19. A merchant marked prints which cost him 7¢ to be sold at 9¢. What % advance on cost was this?

20. Bought 2560 lb. of coffee at 31¢ per pound, and paid \$1.50 per hundred for freight and \$1 for cartage. What % did I make by selling it at 45¢ per pound?

21. By selling cloth at \$5.50 per yd., I make 10 % on the cost? What was the cost?

22. By selling a horse for \$230, I lost 8% on the cost. What was the cost?

23. What was the cost of cloth marked \$3.50 per yd., this being 15% advance on the cost?

24. A merchant having marked down his goods 33⅓% from his usual retail price, which was 20% advance on cost, what was the cost of an article now marked 20¢?

25. A merchant who had marked a certain lot of goods to sell at 15% advance on cost, in consequence of a rise in the market marked them up 5% on the former retail price. At what % advance on cost were they now marked?

26. From what price can I fall 33⅓% on goods which cost \$3.20 per yard, and still make 20%?

27. At what must I purchase nails by the keg (100 lb.) to sell them at 5¢ per lb., and make 15%?

28. I have marked goods which cost me \$2.50 per yd. to sell at 25% advance. What % can I fall on this selling price, and make 20% on the cost?

29. A grocer, by retailing sugar at 12½¢ per lb., made 10% on the cost. What was the cost per bbl. of 200 lb.?

30. If, by selling nails at 6¢ per lb., I lose 4%, will I gain, or lose, by selling at 7¢? What % on cost?

31. At what must I buy boots by the case (1 doz. pairs) to make 15%, and sell at $4.60 per pair?

32. Coffee, which cost me 14¢ per lb., I sell at 6 lb. for $1. What % do I make?

33. If tea at 75¢ per lb. gives a profit of 20%, what would it yield at 56¼¢? What at 50¢? What was the cost?

34. What % does a man make who sells a horse for $100 which was given to him? What, if he sells it for $250?

35. Owning ⅗ of a factory, I sold 16⅔% of my interest for $800, which was considered to be 10% less than its real value. What was the estimated value of the factory?

36. What % is made by buying berries by dry measure, and selling at the same price per quart liquid measure? What % is lost if I buy by liquid measure, and sell by dry, at the same rate per quart?

37. A druggist buys a certain drug at $7.00 per lb. av., and sells it at $1.00 per oz. apothecaries' weight. What % profit does he make?

38. How must an article be sold by the dram (apothecaries'), which cost $5.00 per lb. av., to make 50% profit?

39. I made $1750 in a certain business in 1876, which was 15% more than I made in 1875. How much more did I make in '76 than in '75?

40. If I sell ¾ of an article for the cost of the whole of it, what % gain do I make on the part sold?

If I sell 3 parts for what 4 parts cost, I sell at ¼ more than cost; i.e., at 33⅓% advance.

41. If I sell ¾ of an article for what ½ of it cost me, what % do I lose on the part sold?

If I sell 3 parts (¾) for what 2 parts ($\frac{1}{2} = \frac{2}{4}$) cost me, I sell at ⅔ cost, and hence lose ⅓ cost, or 33⅓%.

Or suppose the article cost me $100: I sell for $50 what cost me

$75. Hence I lose $25, which is $\frac{1}{3}$ the cost of the part sold. ∴ I lose $33\frac{1}{3}\%$.

42. If $\frac{4}{5}$ of the buying price equals the selling price, what is the loss per cent?

43. If $\frac{4}{5}$ of the selling price equals the buying price, what is the gain per cent?

44. B lost 5 per cent by selling a hektoliter of turpentine which cost $15. For what did he sell it a liter?

45. Sold cloth which cost me 8 francs per yard at 8 marks per yard. What % did I make?

COMMISSION, BROKERAGE, AND BANKRUPTCY.

317. An **Agent, Broker,** or **Commission-Merchant,** is a person who does business for another.

Commission or **Brokerage** is the percentage paid an agent, broker, or commission-merchant, and is estimated at a certain rate % on the amount of business done.

The distinction between Commission and Brokerage is not very clearly defined: but in a general way it may be said that the term *Broker* is more exclusively applied to persons dealing in money, stocks, exchanges, or other more characteristically monetary matters; while a Commission-Merchant deals in some other kind of property. The term *Agent* seems to be getting into use as a general term, covering all classes of business-men away from the central office who render service in business-affairs for others.

318. The **Amount** of money *received* or *expended* in behalf of another is usually the *base* on which commission is reckoned, except in case of dealings in stocks and exchange; in which cases it is customary to estimate brokerage on the *par value* of the paper.

319. A Bankrupt is a person who, having failed in business, is unable to pay his debts.

In such cases it is customary for the bankrupt to transfer his property to another person, called an *Assignee*, whose duty it is to settle with the creditors. The *Liabilities* are the sum of the debts. The *Assets* are the value of the property, including money, notes, and accounts due the bankrupt, etc.

1. Mr. Smith left with merchant Jones 5 doz. pr. gloves to be sold at $1.50 per pair, agreeing to allow Mr. Jones 10% for selling. What was Mr. Jones's commission (percentage) on 4 doz. which he sold? How much would he pay over to Mr. Smith?

2. Mr. Smith left with merchant Jones a certain number of doz. pr. gloves to be sold at $1.50 per pair, agreeing to allow him 10% for selling. On settlement, Mr. Jones's commission was $12.60. How many dozens were sold?

3. $13.86 commission for selling $198 worth of goods is what % commission?

4. A dealer in real estate sold a farm for Mr. A., charging him 5%. His commission was $375. For what did he sell the farm?

5. A real-estate dealer charges me 5% for selling my farm of 320 acres at $58 per acre. How much do I receive for the farm?

6. A grain-dealer sells 2000 bu. of wheat for me, and pays me $2450, his commission for selling being 2%. At what price per bushel did he sell it?

7. An attorney collects a claim of $650.50. He pays $23.75 costs, and charges 5% for collection. What does he pay the owner of the claim?

8. A bankrupt's assets were found to be $33000, and his liabilities $86000. What % can he pay? What will a creditor receive whom he owes $650?

9. I receive $850 on a claim of $1250 against a bankrupt estate. What % does the estate pay? If the liabilities are $95000, what are the assets?

10. Paid an attorney \$57.82½ for collecting a claim at 4½%. What was the claim?

11. A commission-merchant charged \$25.50 commission at 2½% for selling 120 bbl. of flour. What did he sell it at per barrel? and how much did he pay over to the owner?

12. What amount of goods can be purchased for \$318 if the agent retain 6% on the amount expended?

13. A real-estate agent received \$3000 to invest in land, after deducting his commission of 6¼%. What amount did he invest?

14. A commission-merchant receives 12600 bu. of wheat, and sells it at \$1.37 per bushel on a commission of 3½%. What was his commission?

15. A commission-merchant charged \$17.28 for selling 640 bu. of potatoes at 60 cents a bushel. What was the rate per cent?

16. A commission-merchant sold 127 bbl. of flour for me at \$7.50 per bbl. He paid \$12.85 freight; and this, with his commission, amounted to \$70. What was the rate per cent of his commission?

17. How many barrels of flour, at \$7 a barrel, can an agent buy for \$441, after taking out his commission of 5%?

18. A country merchant forwarded 160 bbl. of flour to be sold at \$6.25 a barrel, the agent receiving a commission of 3% for selling. After paying \$5.75 for cartage, and deducting his commission of 1½%, he invested the proceeds in plaster at \$19 a ton. How many tons did he buy?

19. I sent a note of \$2500 to a lawyer in Hudson, with instruction to secure what he could upon it, as I understood that the firm against which the note was had gone into bankruptcy. He secured 62½% on the face of the note, and charged me 5% commission. How much did he remit to me?

The base is 62½% of 2500. Why? (See 318.)

20. What amount of goods can be bought for $8758.25, allowing 2½ per cent commission?

21. An agent received $5650 to invest in wheat, at a commission of 3⅓ per cent. How much was expended in wheat? and what was the agent's commission?

22. A Michigan merchant sent to a commission-merchant in Chicago 12 tons of maple-sugar during the season. The Chicago merchant paid railroad charges at 50¢ per cwt., and $3.00 in all for cartage. His commission was 2½ per cent. What would he remit the Michigan merchant, he having sold the sugar at 25¢ per pound?

23. A bankrupt's assets are found to be land valued at $3750; notes and accounts due him, $750; cash, $1250. He owes in New York, Mr. A., $4500; in Toledo, Mr. R. $1575, and Mr. C. $3000; and I have a claim of $2540. Allowing the assignee 5% on the assets, and court expenses $350, what per cent is my claim worth?

STOCKS.

320. A Company is an association of persons for transacting business.

Business Companies are of two general classes, — *incorporated* and *unincorporated*. The former are spoken of as *Corporations*, and the latter as *Firms*, *Houses*, or *Partnerships*.

321. Capital Stock, or *Joint Stock*, is the amount of money paid, together with that subscribed, but not yet paid in, for the purpose of carrying on the business of the company or corporation.

322. Stocks are the certificates of a corporation, signed by the proper officers, and showing that the holder owns so *many shares in* the capital stock of the company.

Stocks are usually reckoned by *Shares* of $100 each. Thus, if the capital of a bank is $200,000, and I own 50 shares of the stock, I own $\frac{5,000}{200,000}$, or $\frac{1}{40}$ of the capital, and hence am entitled to $\frac{1}{40}$ of the net profits of the business. An owner of *stocks* is called a *Stockholder*.

323. The **Gross Earnings** of a company is the total amount of money, or its equivalent, received in the transaction of its business.

324. The **Net Earnings** of a company is the amount that is left after deducting from the gross earnings the expenses of conducting its business, losses, and accrued interest upon its bonds or other obligations.

325. An **Assessment** is a sum required of stockholders to meet the losses or expenses of the company.

326. A **Dividend** is an amount paid out of the *Net Earnings* to the *Stockholders*. It is usually reckoned at a certain per cent on the nominal or face value of the stocks.

327. The **Par Value** of stock is the face of the certificate or bond.

328. The **Market Value** of stock is the price per share at which it can be bought.

329. When stocks sell for more than their par value, they are said to be at a **Premium**; when for less, they are at a **Discount.**

330. **Premium, Discount,** *and* **Brokerage** *are always reckoned on the* **Par Value.**

331. **Stock-jobbing** is the business of buying and selling Stocks and Bonds, with a view to speculation.

332. **Quotations** are public statements made in the newspapers and otherwise of the *market value* of stocks, bonds, etc.

I read in the daily paper to-day the following *Quotations:*—

Kansas Pacific	49½.
Michigan Central	82½.
New-York Central	116½.
Illinois Central	84½.
Western Union Telegraph	105¼.
Wells, Fargo, & Co.'s Express Stock	100.

This means that I can buy stocks in these companies at the annexed rates: i.e., Kansas Pacific at $49½ per *share;* Michigan Central, $82½ per share; New-York Central, $116½ per share, etc. The stock of the 1st, 2d, and 4th are below par, the 3d and 5th above, and the 6th at par.

1. What cost 50 shares Michigan Central Railroad at 49½?

2. At what per cent discount is Illinois Central Railroad stocks, quoted 84½?

3. How much % below par is Kansas Pacific when quotations are 49½?

4. I send to a broker in New York $1000 with which to buy New-York Central stocks, quoted at 116½. If he can buy only whole shares, and charges me brokerage ¼%, how many shares can he buy for me? and how much money will he return? What is his brokerage?

As every share will cost $116½ + $¼ for brokerage, it will cost $116¾. Hence he can buy 1000 ÷ 116¾ = 8 shares and $66 remaining. His brokerage is ¼% of $800, or $2.

5. A gas company whose capital stock is $650000 declares a semi-annual dividend of 4½%. How much will be Mr. A.'s dividend, who holds 20 shares? What was the entire dividend?

6. A railroad corporation whose capital stock is $5000000 wishes to raise by assessment $45000. What will be the rate per cent assessed? and what Mr. Z.'s assessment, who owns 15 shares?

7. How much money must I remit to Chicago to buy 42

shares in Illinois Central, quoted at $84\frac{1}{2}$, allowing the broker there $\frac{1}{3}\%$ for buying?

8. If the Illinois Central declares a dividend of 5% per annum, what % do I make on my investment in Ex. 7?

As I pay $\$84\frac{5}{8}$ for each share, and receive \$5 per annum on a share, the question is, What % of $\$84\frac{5}{8}$ is \$5?

9. If New-York Central pays 4% semi-annually, and the quotation is $116\frac{1}{2}$, what per cent per annum shall I make on my money by investing in it, brokerage for buying being $\frac{1}{2}\%$ (calling 4% semi-annually the same as 8% per annum)?

10. At what rate must I buy stock in a company which pays $4\frac{1}{4}\%$ semi-annual dividends to make the investment yield me 12% per annum, brokerage being $\frac{1}{2}\%$, and calling $4\frac{1}{4}\%$ semi-annually the same as $8\frac{1}{2}$ annually? What would such stock be quoted?

The simplest way to think of this question is this: If I knew what a share cost me, I would *divide* the annual receipt from it, \$8.50, by this cost, and get .12, if I was making 12%. Hence \$8.50 divided by the cost of 1 share is .12; then $\$8.50 \div .12 = \$70\frac{5}{6}$ is the cost of 1 share. But $\$\frac{1}{2}$ of this is required for brokerage. Hence the quotation would be $70\frac{1}{3}$.

11. I have \$2500 to invest for a couple of orphan-girls. Will it be better for them that I invest it in stock quoted at $81\frac{1}{2}$, which pays $2\frac{3}{4}\%$ semi-annual dividends ($5\frac{1}{2}\%$ per annum), brokerage at $\frac{1}{4}$, or in real estate paying 7% per annum?

INSURANCE.

333. Insurance is a branch of business in which companies called *Insurance Companies* make contracts to pay specified sums of money to other parties in the event of certain losses to which the latter may be liable, the company receiving a percentage on the sum guaranteed.

334. The contract is called a **Policy.** The sum which the party insured pays to the company is called the **Premium.**

335. There are two principal departments of the insurance business; viz., *Fire Insurance* and *Life Insurance.*

336. As to the *Constitution of the Company*, Life-Insurance companies may be *proprietary*, *mutual*, or *mixed*, — *Proprietary* when the stock is subscribed, and the company constituted in the ordinary way of organizing business corporations; *Mutual* when each person insured becomes a member of the company, and hence is both insured and insurer; *Mixed* when both features are combined.

337. *Policies* are *Life Policies* when the amount guaranteed is due on the death of the insured; *Term Policies* when this sum is payable upon the death of the insured, provided it occurs within a specified time; *Endowment Policies* when the guaranty is payable when the insured reaches a certain age, or at his death if it occurs before he reaches that age.

338. In addition to these *principal* forms of insurance, there are various others; as *Marine*, *Accident*, *Health*, etc. The character of the ordinary operations and computations will be understood from the following problems. Any attempt to explain the principles upon which life insurance is calculated, or the theory of *probabilities* on which all insurance depends, is quite beyond the scope of this work.

For a clear and simple exposition of the principles on which life-insurance calculations are made, see the author's "SCIENCE OF ARITHMETIC."

1. An insurance company gives me a contract agreeing to pay me $\frac{2}{3}$ the value of my house in the event of its being burned during the year, for which security I pay the company $\frac{5}{8}\%$ on the amount insured. What is the written contract called? My house being worth $3000, how much do I pay for the insurance? What is this called? If my house is burned during the time, how much do I receive?

2. What is the annual premium on a policy which insures a house worth $12000 for $\frac{5}{6}$ its value at $\frac{1}{2}\%$?

3. I insure my house for $3500, furniture for $1500, and library for $1000, at 80¢ per annum on $100, for 3 years, paying $1 for policy, and 75¢ for survey (i.e., examination of premises). What is the total premium?

4. A life-insurance company gives me a contract in which they agree to pay my wife $4000 at my death, in consideration of my paying them an annual premium of 3%. What is my annual payment?

5. What is the annual premium on a life policy for $3500 at 2¾%?

6. A ship was insured for $38000 for 2 months at 4%, and its cargo for $22400 for 3 months at 6½%; and, being at sea the 64th day afterwards, the ship and cargo were destroyed by fire. What sum more than the premium received would the insurance company have to pay?

7. What is the rate % for insurance of $5000 in an accident-insurance company for one day, if the premium paid is 10 cents?

8. The premium for insuring goods was $5.50; the rate was 1½%. For what amount were they insured?

9. If at 8% the premium on a boat was $146, for what amount was the boat insured?

10. If $59.22 was the premium, and 7% the rate, what was the amount of insurance?

11. A premium of $129.60 was paid for the insurance of $17280 on a store. What was the rate % of insurance?

12. What was the rate % of insurance on a box of goods, if it was insured for $340, and the premium was $4.25?

13. Mr. A. insures his house at 5% for $3160, which covers ¾ the value of the house, and $2 which he pays for the execution of the policy. What is the value of the house?

14. A ship from New York, valued at $52650, was insured for a whaling-cruise at 8¼%. What was the premium?

15. A gentleman has a house worth $16500, and furniture

valued at $3675, both of which are insured for $\frac{2}{3}$ of their value at $1\frac{1}{2}\%$. What annual premium does he pay?

16. A, at the age of 40, effects an insurance on his life for 4 years, for the sum of $8000, at the rate of $1.80 on $100 per annum. What is the annual premium?

17. My house is valued at $5000, my furniture at $2000, and my library at $2000. If I get the whole insured for $\frac{2}{3}$ its value at $\frac{3}{8}\%$, what is my annual premium? In the event of damage by fire, if my house is injured to the extent of $1500, my furniture $600, and my library $300, what shall I receive from the company?

ANS. Premium, $22.50. Ordinarily policies make the company liable to pay all damages up to the amount insured. Hence in this case I should receive $2400.

Companies often reserve the right to restore the damaged building in lieu of paying its estimated damage in money to the owner.

18. My house is worth $1200. I have it insured at $\frac{2}{5}\%$, so as to cover $\frac{2}{3}$ of its value and the premium if it chances to burn within the year. What is the sum named in the policy? What the premium?

Of every $1 premium paid, how much applies on the property, and how much to refund premium?

19. A manufacturing establishment, worth $200000, is insured in Co. A for $\frac{1}{5}$ its value at $\frac{3}{4}\%$, in Co. B for $\frac{1}{4}$ its value at $\frac{3}{5}\%$, and in Co. C for $\frac{1}{10}$ its value at $\frac{2}{3}\%$. What is the total annual premium? In case the establishment is damaged by fire to the extent of $15000, what is due from each company?

20. Jan. 1, 1879, I took out a policy on my house, valued at $6000, covering $\frac{3}{4}$ its worth at $\frac{3}{5}\%$, and paid $1 for policy, and $1 for surveying. I carried the policy 6 years and 3 mo., when the house burned. What was my loss, not reckoning interest on the annual premiums?

21. B's house, worth $15880, is insured for $\frac{4}{5}$ of the value at 4%, so as to include the premium if burned. Required the sum stated in the policy.

22. For what sum must a stock of dry-goods, worth $7500, be insured in order to include the value and the premium, when the rate of insurance is $2\frac{1}{2}$%?

TAXES AND DUTIES.

Taxes.

339. A Tax is money required by the government to be paid by the people of the country for the support of government, or for public enterprises.

The general theory of taxes is that they are laid upon *property*, and not upon persons: so it is designed that every dollar's worth of property shall bear an equal part of the tax to be raised. Apportioning the tax to be raised, and determining the value of each person's property, is called *Assessing*.

There are, however, in some of the States, what are called *Poll Taxes*. A poll tax is usually a small sum (75¢ or $1) required of every male over 21 years of age.

1. In a certain school-district, the entire property of which is valued[1] at $125,000, a schoolhouse costing $5000 is to be built by public tax. How much is assessed on a dollar? What is Mr. Smith's schoolhouse tax, his property being assessed at $3000? Mr. Jones's, whose property is assessed at $4500? Mr. White's, whose property is assessed at $10000?

If $125000 worth of property is taxed $5000, $1 bears $\frac{1}{125000}$ of the tax. Hence the tax on $1 is $\frac{5000}{125000}$ of a dollar, or 4 cents. Mr. Smith's tax would therefore be 3000 times 4¢, or $120; Mr. Jones's, $180; and Mr. White's, $400.

[1] Such valuation for the purpose of assessing taxes is often made at half or less than half the actual value of property, though the present tendency is to require that it be made at the cash value.

2. The total State tax of Michigan for 1875 was $903-435.50.[1] The total valuation of property in the State was $630000000. What was the State tax on $1 valuation? What was a man's State tax whose property was valued at $2000?

3. In a certain village the taxes were, for State purposes, 1½ mills on $1; for county purposes, ½ mill on $1; for school, 3 mills; for township purposes, 2 mills; for corporation (village expenses), 2¢. What were a man's taxes whose property was listed (put on the tax-list by the assessor) at $3000? One whose property was listed at $600? What was the school tax of each? What the village tax?

4. A certain State legislature levies $\frac{1}{20}$ of a mill on a dollar as a tax for a particular interest. The valuation being $630000000, what will this yield?

5. In a State in which the entire valuation is $2500-000000, what amount will a tax of .01 of a mill on a dollar raise?

6. In a certain school-district the people desire to raise $5000 for the purpose of building a schoolhouse. It is estimated that 6% of the amount levied will not be collected, and 5% is to be allowed for collecting. What amount must be levied?

What will $1 tax levied net for the building of the house? The collector has 5% on all he collects, including his own percentage.

7. Verify the above. That is, in a certain school-district there was levied $5599.10 tax for building a schoolhouse. Of this 6% was uncollectible, and the collector was allowed 5% for collecting. What did the tax net for the building?

8. A bridge costing $250000 is to be built between two

[1] This was for such objects as the support of the State government; providing for interest and payments on the State debt; for the various State institutions, as the University, Agricultural College, Normal School, State Public School, the various asylums and prisons; and for carrying forward the building of the State Capitol.

cities. The larger city is to bear $\frac{3}{5}$ of the expense. The taxable property of this city being $80000000, what will be a man's bridge tax who is taxed on $15000, assuming that the levy is made to include $3\frac{1}{2}\%$ for collecting, and 10% as uncollectible?

9. Verify the last. That is, having all the facts given, including the answer, except the cost of the bridge, to find the cost of the bridge.

10. The valuation of the taxable property of a town being $2100000, what will a levy of $1\frac{1}{2}$ mills on a dollar net, 8% being uncollectible, and 3% being paid for collecting?

11. In a certain city a man pays $35.22 tax on a valuation of $1050. The entire tax levied being $425000, what is the total valuation of the city property? What is the actual value of the city property, if this man's property is worth $4500?

12. The total valuation in a certain city is $1500000, and a man taxed on $1050 valuation pays $35.22. What is the total tax levied?

13. The net proceeds of a certain assessment was $150000. Allowing $4\frac{1}{2}\%$ of the levy as having proved uncollectible, and $3\frac{1}{2}\%$ commission for collection, what was the total tax levied?

14. The assessed value of property in the State of Illinois for 1873 was $1355401317. The total tax assessed was $21963821.29. Of the latter, $5023609.50 was for State purposes, $5533091.20 for county purposes, $1583942.32 for city purposes, and $9823178.27 for town, district, and other local purposes. The school tax for this year was $999587.91.

15. What was the tax on $1 valuation in the State of Illinois for 1873?

16. From the data in 5 and 6 fill out the following

TABLE.

PROPERTY.	TAX.	PROPERTY.	TAX.	PROPERTY.	TAX.	PROPERTY.	TAX.
$1		$10		$100		$1000	
2		20		200		2000	
3		30		300		3000	
4		40		400		4000	
5		50		500		5000	
6		60		600		6000	
7		70		700		7000	
8		80		800		8000	
9		90		900		9000	

17. From the above table, when filled out, determine by mere addition what a man's tax would be who was taxed on $5680 real estate and $1050 personal property. A man who was taxed on $548 real estate and $85 personal property. A man taxed on $8572 real estate and $12765 personal property.

18. How much property was a man assessed upon who paid $125 tax in Illinois in 1873? How much of this $125 was for State purposes? How much for school tax? If his property was assessed at $\frac{2}{3}$ its real value, what was this man worth?

19. What school tax did a man in Illinois who was worth $20000 pay in 1873, supposing his property to have been assessed at $\frac{1}{2}$ its real value?

United-States Revenue.

340. The expenses of the General Government[1] are provided for by a *Tax on Imported Goods*, and by the *Internal Revenue*.

341. **Duties**, or **Customs**, are taxes levied on imported articles, and are either *Specific* or *Ad Valorem*.

342. *Specific Duties* are duties levied on particular articles irrespective of their value. *Ad Valorem Duties* are duties levied on articles bought in foreign markets, and are estimated at a certain *per cent* on the net cost.

In transporting goods from one country to another there is more or less liability to loss, and consequently there are certain deductions made for such losses in cases in which specific duties are charged. The principal of these are *Draft*, an allowance by weight for waste; *Tare*, a deduction from gross weight, made for the weight of the box, bag, or other thing containing the goods; *Leakage*, a deduction made for actual loss through leakage from casks; *Breakage*, a loss by breakage from things imported in bottles.

In order to collect *Duties*, or *Customs*, Congress determines on what articles and at what rates they shall be charged; and the schedule embracing these facts is called a *Tariff*.[2] Congress also designates *Ports of Entry*; that is, ports where imported goods can be landed, and where *Custom-Houses* are built, and officers of government kept to collect customs.

343. **Internal Revenue** is revenue derived from sale of public lands; from sale of postage and other stamps; from taxes on certain manufactures, as distilled and malt liquors, etc.

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[1] That is, the government of the United States as a whole. These expenses are provided for by Congress, and embrace the salaries of United-States officers, the expenses of the mail, army, and navy service, the improvement of navigation, expenses for national buildings, etc.

[2] From Tarifa, a fortress established by the Moors at the Straits of Gibraltar, where they exacted duties from all vessels entering or leaving the Mediterranean Sea.

Laws regulating the internal revenue are called *Excise Laws*, in distinction from *Tariff Laws*, which regulate *Duties*.

Ex. 1. — M. S. Smith, Detroit, Mich., importer of watches, etc., received an invoice of 3 cases of Swiss watches, costing 22800 francs; duty, 25%; cost of transportation, 35 francs; commission to agent in Geneva, 2½%. What was the total cost?

SOLUTION.

Net cost in Geneva,	22800	francs.
Duties paid in New York,	5700	"
Commission to agent in Geneva,	570	"
Transportation,	35	"
Cost in francs,	29105	
	.193	
Total cost,	$5617.265	

2. A dry-goods importer received at Boston from Liverpool the following invoice: —

650 yd. broadcloth	@	13*s.*
1246 yd. lace	@	2*s.*
1200 yd. coach lace	@	11*d.*
1950 yd. ingrain carpet	@	3*s.*
2560 yd. drugget	@	2*s.* 4*d.*

The duty on the broadcloth, carpeting, and drugget was 30%, and on the laces 25%. What were the customs?

3. What is the duty, at 40%, on 110 chests of tea, each containing 67 lb., and invoiced at 90¢ a lb., the tare being 9 lb. a chest?

4. What is the duty on 50 hhd. of molasses, 63 gal. each, at 20¢ per gal., leakage 3%?

5. What is the duty on 15 casks of Malaga wine, each holding 54 gallons, invoiced at 45 cents per gallon, allowing 2 per cent for leakage, the custom-house rate being 20 cents per gallon, and 25 per cent *ad valorem*.

On some articles both specific and *ad valorem* duties are charged, as is supposed in the last example.

6. A wholesale merchant in Boston imported 80 dozen bottles of Cologne water, invoiced at $5.25 per dozen. Allowing 5 per cent for breakage, and regarding a dozen bottles as equivalent to $2\frac{3}{4}$ gallons, what is the duty, the rate being $3 per gallon, and 50 per cent *ad valorem*?

7. What is the duty on 20 hhd. sugar, invoiced at 1160 lb. each, tare being 10%, and the duty 3¢ per lb.?

8. What is the duty on 3 lots linen hdkfs., which the appraiser classifies as follows? —

1 lot, value £34 4*s.* 6*d.*, duty 35%.
1 lot, " £14 12*s.* 6*d.*, " 40%.
1 lot, " £36 11*s.* 3*d.*, " 40%.

9. How much will it take to pay the duties on 3 cases of French *mousseline de laine*, containing 7563.8 meters, invoiced at .88 francs, the impost being 8¢ per yd., and 40% *ad valorem*.

10. What is the duty on 5 T. 16 cwt. 3 qrs. 20 lb. of steel, invoiced at 25¢ per lb., the duty being 20%?

In the United-States custom-houses 112 lb. is called a hundredweight, whence 28 lb. is a quarter; i.e., the *Long Ton* is in use.

SECTION III.

INTEREST.

Ex. 1. — Mr. Smith lends me $250 for a year, and I agree to pay him back the $250 at the close of the year, and 6% additional for the use of the money. How much do I pay for the use of the money? How much do I pay Mr. Smith in all at the end of the year?

344. Interest is money paid for the use of money.[1]

345. The **Principal** is the sum for the use of which *interest* is paid.

It will be seen that *Principal* corresponds to *Base*, as heretofore used, and *Interest* to Percentage; so, also, the *Amount* is the sum of principal and interest.

346. Simple Interest is interest which is considered as falling due only when the principal is paid, or when a partial payment is made. It is usually reckoned at a certain per cent per annum (year).

According to this principle, — viz., that the interest does not fall due till a payment is made on the principal, — no interest is allowed on accrued interest.

[For information about USURY LAWS, see (**364**)].

2. What is the simple interest on \$125 for 3 yr. at 7% per annum? What the amount?

OPERATION.	EXPLANATION.
\$125 .07 8.75 3 \$ 26.25 125.00 \$151.25	Since 7% is .07 of the principal, the interest for 1 yr. is \$125 × .07, or \$8.75; and the interest for 3 yr. is 3 times the interest for 1 yr., or \$8.75 × 3 = \$26.25. The amount, being the sum of principal and interest, is \$151.25.

3. What is the interest on \$250.60 for 2½ yr. at 10% per annum? What the amount?

4. If I borrow of Mr. White \$325 for 1 yr. 8 mo. 15 da. at 7%, what shall I have to pay him at the expiration of the time?

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[1] As the basis on which interest is computed is always money, it is not deemed best to cumber the definition with any allusion to any thing else.

EXPLANATION.

Since 7% is .07 of the principal, the interest for 1 yr. is $\$325 \times .07 = \$22.75$. For 6 mo. the interest is $\frac{1}{2}$ of the interest for a year, and the interest for 2 mo. is $\frac{1}{3}$ the interest for 6 mo. 15 da. is $\frac{1}{2}$ of 1 mo., or $\frac{1}{4}$ of 2 mo. Hence the interest for 15 da. is $\frac{1}{4}$ the interest for 2 mo. Adding these results, we have the interest for 1 yr. 8 mo. 15 da.

OPERATION BY ALIQUOT PARTS.

$325	
.07	
$22.75	Int. for 1 yr.
11.375	Int. for 6 mo. $= \frac{1}{2}$ int. for 1 yr.
3.792	Int. for 2 mo. $= \frac{1}{3}$ int. for 6 mo.
.948	Int. for 15 da. $= \frac{1}{4}$ int. for 2 mo.
$38.86	Int. for 1 yr. 8 mo. 15 da.
325.00	Principal.
$363.86	Amount.

OPERATION BY DECIMALS.

30	15.	da.
12	8.5	mo.
	$1.7\frac{1}{2}$	yr.

$325
.07
22.75
$1.7\frac{1}{2}$
189
15 925
22 75
38.864
325
$363.86

The interest for 1 year is $22.75, and 1 yr. 8 mo. 15 da. $= 1.7\frac{1}{2}$ yr. Hence the interest for 1 yr. 8 mo. and 15 da. is $\$22.75 \times 1.7\frac{1}{2}$.

To Compute Simple Interest.

347. Rule. — *Multiply the principal by the rate, and this product by the time in years.*

To find the amount, add the interest to the principal.

While this rule is perfectly general, some modifications in the form of the work arise from different methods of treating the *months* and *days*. We give three: 1st, Considering the months as aliquot parts of a year, and the days as aliquot parts of a month (30 da.); 2d, Reducing the months and days to decimals of a year; and, 3d, Calling the months 12ths of a year, and the days 30ths of 12ths, *indicating the operations, and cancelling.* (See **348, 349.**)

The method of computing interest by aliquot parts is in more general use, notwithstanding that the method by decimals usually requires less work. The most expeditious method is by cancellation. (See **348, 349.**)

Business-men, who have frequently occasion to compute interest, — as bankers, — *generally make use of Tables.*

5. What is the amount of $350 for 3 yr. 10 mo. 19 da. at 8%?

By Aliquot Parts.

$350	
.08	
28.00	
3	
84.00	
14.00	
7.00	
2.3333+	
1.1666+	
.2333+	
.0777+	
$108.81	Interest.
350.00	
$458.81	Amount.

By Decimals.

30 | 19.		$350
12 | 10.633+		.08
3.886+		$28.00
28		
31 088		
77 72		
$108.808	Interest.	
350		
$458.81	Amount.	

Solve the following by each of the above methods, and observe which is the more expeditious. Find both interest and amount.

6. $52.80 at 6% for 2 yr. 8 mo. 12 da.
7. $235.50 at 10% for 3 yr. 6 mo. 10 da.
8. $245.60 at 8% for 2 yr. 7 mo. 21 da.
9. $500 at 6% for 2 yr. 5 mo. 12 da.
10. $750.50 at 7% for 1 yr. 8 mo. 20 da.
11. $436.75 at 5% for 1 yr. 2 mo. 15 da.
12. $230 at 6% for 11 mo. 15 da.
13. $1385.50 at 15% for 23 da.
14. $14.30 at 8% for 2 yr. 9 mo.
15. $325.25 at 6½% for 2 yr. 9 mo. 12 da.
16. $2360.25 at 8% for 7 mo.
17. $18.28 at 5% for 5 yr. 9 da.
18. $87.50 at 7% for 3 yr. 3 mo.
19. $480 at 15% for 6 yr. 3 mo.
20. $18.20 at 5¾% for 9 yr. 9 mo. 9 da.

348. The last example solved by cancellation:—

Int. for 9 mo., $\frac{1.05 \times \overset{3}{\cancel{9}}}{\underset{4}{\cancel{12}}} = \frac{3.15}{4} = 79\text{—}$

Int. for 9 da., $\frac{1.05 \times \overset{3}{\cancel{9}}}{\cancel{12} \times \cancel{30}_{10}} = 0.26+$

For the parts of 1 yr. the interest will be found with sufficient accuracy if we take the interest for 1 yr. to the nearest cent.

	\$18.20
	.05¾
	910
	455
	9100
Int. 1 yr.,	1.0465
	9
Int. 9 yr.,	9.4185
" 9 mo.,	.79
" 9 da.,	.026
	\$10.23

349. Rule. — *To find the interest for months by cancellation, write as the numerator of a fraction the interest for 1 year, taken to the nearest cent, and the number of months as factors. For the denominator write 12, and then cancel.*

For days write as the numerator the interest for 1 year, and the number of days in the same way, and for the denominator 12 × 30, and then cancel.

That the three methods of considering the time are all embraced under the general rule (**347**) may be shown by indicating the three methods of solving the last problem thus:—

PRINCIPAL.	RATE.	TIME.	INTEREST.
\$18.20	× .05¾	× (9 + ½ + ½ of ½ + $\frac{1}{10}$ of ¼)	= \$10.23.
\$18.20	× .05¾	× (9.7¾)	= \$10.23.
\$18.20	× .05¾	× (9 + $\frac{9}{15}$ + $\frac{9}{12 \times 30}$)	= \$10.23.

These multiplications by the time in the first two methods would appear in the practical work thus (for the 3d, see above):—

Int. for 1 yr.,	1.0465	1.0465
	9	9.7¾
	9.4185	5232
Int. for ½ yr.,	.5232	2616
" ½ of ½ yr.,	.2616	73255
" $\frac{1}{10}$ of ¼ yr.,	.0261	94185
	\$10.23	\$10.23

21. $64.50 at 7% for 2 yr. 16 da.
22. $725 at 3½% for 5 yr. 2 mo. 18 da.
23. $5000 at 7% from May 6, 1875, to July 7, 1877.

Find the time by subtracting dates (**265**).

24. $81.25 at 6% from Aug. 6, 1873, to Nov. 4, 1876.
25. $105.23 at 10% from June 10, 1871, to Oct. 1, 1875.
26. $76.42 at 5% from May 9, 1874, to Aug. 9, 1874.
27. $18.00 at 8% from Aug. 8, 1875, to Aug. 30, 1875.
28. $5600 at 4½% from April 1, 1876, to April 1, 1878.
29. $43.60 at 3¾% from July 12, 1874, to June 1, 1877.
30. $150.30 at 10% from May 8, 1875, to Nov. 6, 1876.
31. $400 at 10% from Sept. 6, 1876, to Sept. 6, 1878.
32. $350 at 12% from Nov. 9, 1877, to Dec. 9, 1877.
33. $820 at 7% from Dec. 5, 1876, to July 24, 1877.
34. $1000 at 7% from Feb. 7, 1870, to Aug. 9, 1876.
35. $125.41 at 7% from July 10, 1873, to June 10, 1874.
36. $93.25 at 10% from Jan. 1, 1877, to July 1, 1877.
37. $48.50 at 5% from Oct. 3, 1877, to Jan. 3, 1878.
38. $150.40 at 7% from May 23, 1876, to Oct. 1, 1878.
39. $741.50 at 5½% from Nov. 29, 1875, to Aug. 30, 1877.
40. $13.50 at 10% from May 7, 1876, to Sept. 10, 1876.
41. $250 at 6% from July 1, 1873, to April 1, 1874.
42. $450 at 5% from Aug. 7, 1875, to Aug. 7, 1877.
43. $158.23 at 8% from Dec. 25, 1877, to Sept. 23, 1878.
44. $354.40 at 8% from June 30, 1870, to Nov. 1, 1876.
45. $700 at 9% from May 20, 1873, to March 6, 1875.
46. $60 at 12½% from March 17, 1875, to April 6, 1877.
47. $4000 at 5% from Jan. 25, 1872, to Feb. 18, 1874.
48. $250 at 10% from March 6, 1872, to April 30, 1873.
49. $175.50 at 7% from Feb. 7, 1876, to Aug. 11, 1878.
50. $300 at 8% from July 1, 1876, to Jan. 16, 1878.

[For further exercises and various forms of *notes*, see p. 263 *et seq.*]

To Find the Simple Interest on any Principal by means of Interest Tables.

There are several different volumes of such tables in use by bankers and accountants; but the general principle is the same. We have space to give only one page of such tables, and select that which gives the simple interest on \$1, for any time less than 6 years, at 5%, 6%, 7%, 8%, 10%, and 12%. Such volumes generally contain tables which enable us to take the interest on any sum directly from the table, requiring no arithmetical process but addition.

350. Rule. — *To find the interest on any sum from the following table, take from the table the interest on* \$1 *for the given number of years, months, and days, and add these results. Multiply this sum by the given principal.*

1. Find from the table the simple interest on \$143.25 for 3 yr. 7 mo. 22 da. at 7% per annum.

Interest on \$1.		The interest on \$143.25 is $143\frac{1}{4}$ times the interest on \$1: hence .255
.21	For 3 yr.	$143\frac{1}{4}$
.0408	For 7 mo.	64
.0043	For 22 da.	765
.255	For 3 yr. 7 mo. 22 da.	1020
		255
		\$36.53 Int. required.

Solve the following by the table on the next page, finding both interest and amount: —

2. \$340 at 5% for 2 yr. 5 mo. 11 da.
3. \$28 at 10% for 93 da. (3 mo. 3 da.)
4. \$12.50 at 8% for 63 da.
5. \$135.37 at 7% for 5 mo. 13 da.
6. \$81.40 at 8% for 1 yr. 17 da.
7. \$471 at 10% for 2 yr. 6 mo. 5 da.
8. \$251.13 at 7% for 30 da. For 1 yr. 3 mo.
9. \$125.10 at 12% for 340 da. For 3 yr.
10. \$2000 at 10% for 2 yr. For 3 yr. 6 mo. 10 da.
11. \$57.35 at 7% for 2 yr. 8 mo. 10 da.

12%.	6%.	7%.		10%.	5%.	8%.
			YEARS.			
.12	.06	.07	1	.10	.05	.08
.24	.12	.14	2	.20	.10	.16
.36	.18	.21	3	.30	.15	.24
.48	.24	.28	4	.40	.20	.32
.60	.30	.35	5	.50	.25	.40
			MONTHS.			
.01	.005	.00583	1	.00833	.00416	.00666
.02	.01	.01166	2	.01666	.00833	.01333
.03	.015	.01750	3	.02500	.01250	.02000
.04	.02	.02333	4	.03333	.01666	.02666
.05	.025	.02916	5	.04166	.02083	.03333
.06	.03	.03500	6	.05000	.02500	.04000
.07	.035	.04083	7	.05833	.02916	.04666
.08	.04	.04666	8	.06666	.03333	.05333
.09	.045	.05250	9	.07500	.03750	.06000
.10	.05	.05833	10	.08333	.04166	.06666
.11	.055	.06416	11	.09166	.04583	.07333
			DAYS.			
.00033	.00016	.00019	1	.00027	.00013	.00022
.00066	.00033	.00038	2	.00055	.00027	.00044
.00100	.00050	.00058	3	.00083	.00041	.00066
.00133	.00066	.00077	4	.00111	.00055	.00088
.00166	.00083	.00097	5	.00138	.00069	.00111
.00200	.00100	.00116	6	.00166	.00083	.00133
.00233	.00116	.00136	7	.00194	.00097	.00155
.00266	.00133	.00155	8	.00222	.00111	.00177
.00300	.00150	.00175	9	.00250	.00125	.00200
.00333	.00166	.00194	10	.00277	.00138	.00222
.00366	.00183	.00213	11	.00305	.00152	.00244
.00400	.00200	.00233	12	.00333	.00166	.00266
.00433	.00216	.00252	13	.00361	.00180	.00288
.00466	.00233	.00272	14	.00388	.00194	.00311
.00500	.00250	.00291	15	.00416	.00208	.00333
.00533	.00266	.00311	16	.00444	.00222	.00355
.00566	.00283	.00330	17	.00472	.00236	.00377
.00600	.00300	.00350	18	.00500	.00250	.00400
.00633	.00316	.00369	19	.00527	.00263	.00422
.00666	.00333	.00388	20	.00555	.00277	.00444
.00700	.00350	.00408	21	.00583	.00291	.00466
.00733	.00366	.00427	22	.00611	.00305	.00488
.00766	.00383	.00447	23	.00638	.00319	.00511
.00800	.00400	.00466	24	.00666	.00333	.00533
.00833	.00416	.00486	25	.00694	.00347	.00555
.00866	.00433	.00505	26	.00722	.00361	.00577
.00900	.00450	.00525	27	.00750	.00375	.00600
.00933	.00466	.00544	28	.00777	.00388	.00622
.00966	.00483	.00563	29	.00805	.00402	.00644

12. $145 at 8% for 3 yr. 11 mo. 5 da.

13. $280 at 6% for 7 mo. 16 da.

14. A note[1] of $65.80, dated Feb. 20, 1868, and bearing interest at 7%, was paid June 25, 1870. What was the amount paid?

Find the time by subtracting dates (265).

15. On the 21st day of January, 1874, for value received, I promise to pay to John Jones, or order,[2] $350, with interest at 7% per annum.

AUBURN, Dec. 5, 1869. HENRY FISH.

What was the amount of this note Jan. 21, 1874?

16. One day after date, for value received, I promise to pay John Smith, or bearer,[3] one hundred and twenty-five and $\frac{25}{100}$ dollars, with interest at 10%.

ROCHESTER, MICH., May 6, 1875. HENRY HOYT.

What was the amount of this note March 5, 1877?

17. Jan. 6, 1877, for value received, I promise to pay Enos Ames[4] five hundred and fifty dollars, with interest at 7%.

PERRYSBURG, O., May 7, 1875. M. C. PETERS.

What is the amount of this note Jan. 6, 1877?

18. Due Charles Minton, or order, thirty-six dollars, with interest at 6%, value received.

WESTON, O., April 6, 1874. JOHN PIPER.

What was the amount of this due bill Jan. 15, 1876?

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[1] A *note* is a written contract by which one party agrees to pay another party a specified sum.

[2] The words "or order" in this connection prevent John Jones from selling the note, without putting his name on it; i.e., indorsing it. When indorsed, it is said to be "negotiable;" and John Jones can be made to pay it, if Henry Fish does not.

[3] This note is negotiable without being indorsed. Anybody can collect it who may chance to have it. But, if John Smith or anybody else does indorse it, the indorser becomes liable for it.

[4] As this note is payable to nobody but Enos Ames, no one else can collect it. It is not "negotiable," and Enos Ames cannot sell it even by indorsing it. This is the common law. There are statutes in some States, as in Illinois, making such paper negotiable by indorsement.

19. One day after date, for value received, we jointly and severally agree to pay Sarah Miner, or order, seven hundred dollars, with interest at 7%.

SOLOMON PIKE.

CHICAGO, ILL., June 6, 1875. JAMES NOAH.

What was the amount of this note March 29, 1877?

Such a note as the above is called a "*Joint and Several*" note, and either signer is equally liable for it. The holder may take his choice as to which he will collect it from, or he may proceed against both signers.

20. Two years from date, for value received, I promise to pay Stephen Ely, or order, three hundred and seventy-five dollars, with interest. (See **865.**)

PLYMOUTH, MICH., June 7, 1875. SMITH PHILLIPS.

What was the amount of this note June 7, 1877?

What would the amount have been if the note had been dated Plymouth, Mass.? If in Wisconsin? If in Ohio? In Illinois? In Minnesota? In Iowa?

21. Sold my house and lot Aug. 21, 1875, for $5500, receiving $2500 cash, and a 7% note for 3 yr. secured by mortgage for the balance. I immediately let the $2500 at 10% for 3 yr. When both became due, I bought a house and lot for $8560. How much money besides the avails of the house and lot sold did I have to raise?

22. Bought a bill of goods amounting to $750, $\frac{1}{3}$ payable in 30 da., $\frac{1}{3}$ in 60 da., and $\frac{1}{3}$ in 90 da., at 6%. What was the entire cost of the goods?

23. What is the amount of $83.25 at 8% from May 6, 1861, to Nov. 10, 1870?

At 10% from July 8, 1871, to April 17, 1873?

At $6\frac{1}{2}$% from Sept. 13, 1870, to Feb. 13, 1875?

Other Methods of Reckoning the Time.

351. The foregoing method of obtaining the time by subtracting the earlier date from the later, calling 12 mo. a year and 30 da. a month, though the method in common use, does not usually get the exact time when months and days are involved. There are *three* other methods in use: —

1st, That which requires that a day be reckoned $\frac{1}{365}$ part of a year, and that the time be reckoned in years and days, the month unit being excluded. All interest transactions with the United-States Government are computed on this basis. It is called *Exact Interest.*

2d, A number of States have statutes requiring that the time be reckoned in *calendar* years and months, and that the days in excess be reckoned as 30ths of a month.

3d, *Bankers*, and sometimes other business-men, reckon interest on *short-time paper* by taking the exact number of days, and calling them 360ths of a year.

There is never more than one or two days' difference between the time as found by the *common method* and by the *second* above given.

From the interest obtained by the *Banker's Method* for a given number of days, $\frac{1}{73}$ must be subtracted to give the *Exact Interest*, and to the latter $\frac{1}{73}$ must be added to produce the former.

1. What is the exact interest on \$450 at 10% from May 25, 1868, to Jan. 8, 1869? What by the Banker's Method?

The interest for 1 yr. is \$45, and the exact time 228 da. Hence

$$\text{The } \textit{Exact Interest} \text{ is } \frac{\overset{9}{\cancel{\$45}} \times 228}{\underset{73}{\cancel{365}}} = \frac{2052}{73} = \$28.11-.$$

$$\text{By the } \textit{Banker's Method}, \frac{\overset{114}{\cancel{\$45}} \times \cancel{228}}{\underset{4}{\cancel{360}}} = \$28.50.$$

352. Days of Grace. — When money is borrowed at a

bank, the interest is required in advance (i.e., when the money is borrowed), and is reckoned for *three days* (in Pennsylvania four days) more than the nominal time. These three days are called DAYS OF GRACE. The interest in such a case is usually called *Discount*. (See **371.**)

This custom of allowing days of grace has become well-nigh universal with reference to *business paper* (obligations for the payment of money). The general rule is, that a suit at law cannot be instituted for the collection of any such paper until three days after its nominal maturity. Hence, in discounting such paper, it has become customary to compute the amount including these days. When the interest is paid at the time of settlement, it is, of course, reckoned to the date of such settlement.

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353. The Maturity of a note is 3 days after it is nominally due.

When the date at which a note *falls due* is specified, the days of grace are always included.

2. What is the exact interest and what the bank discount on $140.40 from Aug. 29, 1864, to Nov. 28, 1864, at 6%?

Time 60 da.

$$\frac{\$140.40 \times 6 \times 60}{365 \times 100} = \frac{16.848 \times 6}{73} = \frac{101.088}{73} = \$1.38+ \text{ Exact Interest.}$$

(2.808)

All the work appears here except the last division.

$$\frac{\$140.40 \times 5 \times 60}{360 \times 100} = \$1.40, \text{ Banker's Discount.}$$

3. What is the exact interest on $1580 from June 10, 1874, to Feb. 17, 1875, at 10%?

4. What is the exact interest on a $1000 United-States bond, at 5%, from Oct. 1 to May 6 following? From March 13 to Dec. 12 following?

Compute the exact interest on the following, and find the amounts: —

	Date.	Principal.	%.	When Due.
5.	May 10, 1876,	$45.25,	7,	Aug. 8, 1877.
6.	Sept. 20, 1876,	$82.10,	8,	June 5, 1877.
7.	Feb. 10, 1876,	$125.80,	5,	May 11, 1877.
8.	Jan. 1, 1871,	$530.00,	4½,	Nov. 10, 1873.
9.	April 7, 1874,	$1000.00,	4,	July 17, 1876.
10.	Aug. 13, 1876,	$250.00,	4,	Mar. 19, 1877.
11.	May 1, 1876,	$125.00,	7,	Sept. 6, 1876.
12.	Aug. 17, 1875,	$35.50,	10,	Sept. 21, 1875.

13. By the banker's method, what is the interest on a 7% $350 note dated May 11, 1876, and *nominally* payable Sept. 10, 1876? What by the common method? What by the method of exact interest? (All with grace.)

The interest for 1 yr. is $24.50.

By the *Banker's Method* the time is 125 da., or $\frac{125}{360}$ of a year, and the interest is $8.51—.

By the *Common Method* the time is 4 mo. 2 da., and the interest is $8.30+.

By the *Exact Method* the time is 125 da., or $\frac{125}{365}$ of a year, and the interest is 8.39+.

14. For value received, I promise to pay George Van Horn, or order, $500, Nov. 6, 1877, with interest at 10%.

Pontiac, Mich., June 1, 1877. Amos White.

What is the interest on this note by the Banker's Method? When does the note *mature?*

Other Methods.

354. The 6% Method. — When the time is to be reckoned in the common way (i.e., 12 mo. = 1 yr., and 30 da. = 1 mo.), *call* $\frac{1}{2}$ *the number of months* cents, *and* $\frac{1}{6}$ *the number of days* mills, and the sum will be the interest on $1 for the given time at 6%.

The reason for this is evident; since at 6% the interest on $1 for

1 yr. is 6c., or ½c. per mo. Again: as the interest on $1 for 1 mo. is 5 *mills*, it is 1 *mill* for every 6 days.

Ex. What is the interest on $245.50 at 6% for 2 yr. 7 mo. 21 da.?

½ the months is 15.5, and ⅙ the days 3.5. Hence the interest on $1 for the time at 6% is $0.1585. Multiplying this by 245½ gives $38.91, the interest required.

NOTE. —Having the interest at 6%, that at 5% can be obtained by *deducting* ⅙ of the interest at 6%, at 7% by *adding* ⅙, at 4% by *deducting* ⅓, at 8% by *adding* ⅓, etc.

355. THE 1% METHOD. — *Remove the decimal point in the principal 2 places to the left. Multiply this result by the rate per cent, and the time.*

Moving the decimal point 2 places to the left gives the interest on the principal for 1 yr. at 1%. Multiplying this by 7 gives it for 7%, etc.

Ex. Solve the above example in this way: —

Interest for	1 yr., at 1%,	$2.455
		6
"	" 1 yr., at 6%,	$14.73
"	" 2 yr.,	29.46
"	" 6 mo.,	7.365
"	" 1 mo.,	1.227
"	" 10 da.,	.409
"	" 10 da.,	.409
"	" 1 da.,	.041
"	" 2 yr. 7 mo. 21 da.,	$38.91

BY DECIMALS. — The time is 2.64⅙ year. Hence we multiply the interest for 1 yr. by the number of years.

$14.73
2.64⅙
245
5892
8838
2946
$38.9117

356. THE 12% METHOD. — *Remove the decimal point 2 places to the left, and then multiply by the time in months.*

This gives the interest for 12%, which is readily changed to any other rate. Thus, for 6%, take $\frac{1}{2}$ the interest at 12%; for 8%, deduct $\frac{1}{3}$; for 9%, deduct $\frac{1}{4}$; for 10%, deduct $\frac{1}{6}$; for 7%, add $\frac{1}{6}$ to $\frac{1}{2}$.

The reason for this rule is, that 12% is 1¢ on a dollar of principal for 1 mo.

Ex. — The last example computed in this way appears as in the margin.

\$2.455
31.7
17185
2455
7365
2)77.8235
\$38.91

This is an excellent method. It will be observed that all the work which needs to be written for this solution appears in the margin.

[Practice in the use of any of these methods can be secured by the foregoing examples, if desired.]

COMPOUND INTEREST.

357. Compound Interest is interest considered as falling due at regular intervals of time, and to be reckoned as increasing the interest-bearing debt from such times.

This method of reckoning interest allows interest on interest accrued; and hence the term *compound*, meaning *interest on interest.*

1. What is the amount of \$350 at annual compound interest for 3 years at 7%?

OPERATION.

\$350	1st Prin.
.07	
24.50	Int. for 1 yr. on 1st Prin.
350.00	1st Prin.
\$374.50	Amount for 1st yr., or 2d Prin.
.07	
26.2150	Int. on 2d Prin.
374.50	2d Prin.
\$400.715	Amt. of 2d Prin. for 1 yr., or 3d Prin.
.07	
28.05005	Int. on 3d Prin.
400.715	3d Prin.
\$428.76	Amt. at end of 3d yr.

EXPLANATION. — As the interest is considered as falling due at the end of each year, at the end of the first year the debt is $374.50. This is, therefore, to be on interest for the next year. Again: as the interest on this for a year, $26.215, falls due at the end of the year, it is added to the principal for this year, and makes the interest-bearing sum for the 3d year $400.715. This sum on interest for a year amounts to $428.77; which is, therefore, the amount of $250 on compound interest for 3 yr. at 7%.

2. What is the amount of $152 at semi-annual compound interest for 2 years at 6% per annum?

EXPLANATION.

As the interest is considered as falling due at the end of each ½ year, we compute the interest for ½ a year, and then add it to the principal, thus making a new principal for the next ½ year. Instead of multiplying by .06 in this instance, and dividing the product by 2 to get the interest for ½ a year, we simply multiply by .03, which gives the same result.

OPERATION.

$152
.03
4.56
152
156.56
.03
4.6968
156.56
161.257
.03
4.83771
161.257
166.095
.03
4.98285
166.095
$171.08

358. Rule. — *To compute Compound Interest, reckon the interest on the principal for the first interval of time, add it to the principal, and consider this as a new principal for the next interval, etc.*

Or, *Find from the interest tables the amount of* $1 *for the given rate and time, and multiply this by the given principal.*

The result thus found is the AMOUNT. *The Compound Interest is the remainder after the first principal is subtracted from this amount.*

COMPOUND INTEREST TABLE.

YEAR.	3%.	4%.	4½%.	5%.	6%.	7%.
1	1.030000	1.040000	1.045000	1.050000	1.060000	1.070000
2	1.060900	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.092727	1.124864	1.141166	1.157625	1.191016	1.225043
4	1.125509	1.169859	1.192519	1.215506	1.262477	1.310796
5	1.159274	1.216653	1.246182	1.276282	1.338226	1.402552
6	1.194052	1.265319	1.302260	1.340096	1.418519	1.500730
7	1.229874	1.315932	1.360862	1.407100	1.503630	1.605781
8	1.266770	1.368569	1.422101	1.477455	1.593848	1.718186
9	1.304773	1.423312	1.486095	1.551328	1.689479	1.838459
10	1.343916	1.480244	1.552969	1.628898	1.790848	1.967151

3. Find the amount of \$243.12 at annual compound interest for 3 yr. at 4%, both with and without the use of the table. Also the interest.

BY THE TABLE. — The amount of \$1 at 4% for 3 yr. is \$1.12486. Now, \$243.12 amounts to 243.12 times as much as \$1, or \$273.48.

Hence the interest is \$273.48 — \$243.12 = \$30.36.

\$1.12486
243.12
224972
112486
337458
449944
224972
\$273.4759632

Find the compound interest of the following sums for the respective times and rates, both by the use of the table and without it: —

4. \$340 for 2 yr., compounded semi-annually, at 6%.[1]
5. \$100 for 7 yr. at 4½%.
6. \$230 for 6 yr. at 8%. At 5%. At 4%.
7. \$125 for 3 yr., compounded quarterly, at 12%.
8. \$270 for 4 yr., compounded semi-annually, at 8%.
9. \$250 for 3½ yr., compounded semi-annually, at 10%.
10. What is due on a note of \$200, bearing semi-annual compound interest at 9%, 2 yr. 10 mo. from date?

[1] This means, "at 6% per annum," but compounded (i.e., interest added to principal) every 6 mo.: hence it is the same as 3% for 4 years, or \$42.67.

SUGGESTION. — For 2½ yr. the amount is $249.2364. This is then on interest for 4 mo., which makes the whole amount $256.71.

11. What is the difference between the simple interest of $500 at 10% for 3 years and the compound interest on the same sum for the same rate and time?

12. What is the amount of $325 at quarterly compound interest, at 2% per quarter, for 2 yr. 5 mo. 10 da.?

13. What is the interest of $540.20, interest compounded semi-annually, at 5% per semi-annum, for 4 years? What is the difference between this and the interest compounded annually at 10%?

14. What is the difference between the interest of $100, compounded quarterly at 6% per annum, for 2 yr., and the simple interest of the same sum for the same time at 7%?

15. What is the compound interest of $480, at 5% per annum, from May 6, 1873, to July 13, 1875?

16. What is the amount of a note for $500 Jan. 15, 1877, which draws 8% semi-annual compound interest, and is dated Aug. 18, 1874?

The laws of the States usually do not allow the collection of compound interest. In some States such notes as the above would be collectible with simple interest, while in others the taking of such a note would forfeit all interest; and in other States it would entail a still heavier loss, in some even the entire debt.

ANNUAL, SEMI-ANNUAL, AND QUARTERLY INTEREST.

359. Contracts are often made in which it is agreed that the interest shall be paid annually, semi-annually, or even quarterly. This is, in fact, compounding the interest thus often; but, if the payments of interest are not made as they fall due, the general rule is that only simple interest can be collected, although the statutes of some of the States allow simple interest on *the deferred payments of such interest.*

1. On a note for $150, bearing interest at 7%, payable

annually, the debtor had neglected to pay the interest for 3 yr. Allowing simple interest on the deferred payments, what was then due on the note?

As the first year's interest, \$10.50, was not paid when due, it was subject to 2 yr. interest; and in like manner the 2d year's interest on principal was subject to 1 year's interest. Hence we have 3 years' interest on the interest of the principal for 1 year (\$10.50) as the interest upon interest.

OPERATION.

\$150	
.07	
\$10.50	
.07	
.7350	
3	
\$2.205	Int. on int.
31.50	Int. on prin.
150.00	
\$183.71+	Amt.

2. On the same principle as in the last, what is due on a note for \$525, bearing 6% interest, payable annually, the interest payments having been deferred 4 years?

3. On the same principle, what is due on a \$275 10% note, interest payable semi-annually, but deferred 3 yr. 8 mo. 17 da.?

4. As above, what is due on a \$100 8% note, interest payable quarterly, but deferred 1 yr.?

5. As above, what is due on a note for \$200, interest payable annually at 10%, but deferred 10 years?

In this there are 45 yr. interest on the interest for 1 yr.

Find the amount due on the following sums at the respective rates and times, the interest payable as indicated, but considered as deferred: —

6. \$350, 7%, annually, for 3 yr. 5 mo. 10 da.
7. \$820, 5%, semi-annually, for 3 yr. 9 mo. (See below.)
8. \$85.30, 6%, semi-annually, for 1 yr. 10 mo.
9. \$250, 4½%, annually, for 5 yr. 8 mo. 12 da.
10. \$500, 10%, semi-annually, for 2 yr. 7 mo.

The 7th gives 2½% interest on \$20.50 for $6\frac{1}{2}+5\frac{1}{2}+4\frac{1}{2}+3\frac{1}{2}+2\frac{1}{2}+1\frac{1}{2}+\frac{1}{2}=24\frac{1}{2}$ periods of 6 mo. each; i.e., \$12.56 as the *interest on the interest*. To this add the amount of the \$820 at simple interest for the entire time, and we have \$973.75 + \$12.56 = \$986.31.

PARTIAL PAYMENTS.

United-States-Court Rule.

360. It frequently happens that a debtor does not pay his note all at one time. In such a case, whatever is paid at any time is indorsed (credited) on the back of the note, and is called a **Partial Payment** (or simply a payment).

There are several methods in more or less general use for computing interest on such notes. There are, however, but three that are in general use; viz., the one adopted by the U. S. Court and by most of the States, and that called the *Merchant's Rule*, which is much used by business-men for short-time paper, and the *Vermont Rule* for notes bearing interest payable annually.

The *U. S. Court Rule* is based on the two following principles: —

1. *The principal cannot be diminished until the accrued interest is paid.*

2. *Interest shall not draw interest.*

U. S. Court Rule for Computing Interest on Notes on which Partial Payments have been made.

361. Rule. — I. *Compute the Interest on the Principal from the date of the note to the time of the first payment. If this payment equals or exceeds this interest, find the amount, and subtract the payment. Treat this remainder as a New Principal, and proceed to the next payment. Continue the process till the time of settlement is reached.*

II. *If any payment is less than the accrued interest, add such payment to the next, and treat the sum as one payment made at the latter date.*

1. $350.

One day after date, for value received, I promise to pay

John Jay, or bearer, three hundred and fifty dollars ($350), with interest at 7% per annum.

ROCHESTER, N.Y., May 7, 1868. AMOS AMES.

On this note there were the following indorsements: —

Sept. 17, 1870, $100;
Feb. 10, 1872, $50.

How much was due on the note Oct. 25, 1872?

Interest at time of 1st payment	$57.85
As the payment ($100) exceeds this	350.00
we find the amount	$407.85
and subtract the payment	100.00
New Principal	$307.85
Interest on new principal at the time of the 2d payment	30.11
As the payment ($50) exceeds this, we find the amount	$337.96
and subtract the payment	50.00
2d New Principal	$287.96
Amount of this 2d new principal from date of last payment to Oct. 25, 1872	$302.24

2. $475. CHICAGO, ILL., Sept. 14, 1869.

On July 12, 1875, for value received, I promise to pay Peter Price, or order, four hundred and seventy-five dollars ($475), with interest at 6% per annum.

JAMES WHITE.

On this note were the following indorsements: —

April 12, 1871, $25; Aug. 20, 1873, $150;
Nov. 27, 1874, $100; May 1, 1875, $5.

What remained due on this note July 12, 1875?

SUGGESTIONS. — At the time of the 1st payment, the accrued interest was $44.97. If this be added to the principal, and the payment, $25, subtracted, *part* of this interest would be included in the *New Principal*, and hence interest would draw interest, and the 2d principle be violated. Hence we consider the 2d payment, made Aug. 20, 1873, as $175, and compute the interest on the face of the note, $475, up to this date.

This is $112.10; and, the payments ($175) being greater than the accrued interest, we find the amount, $587.10, and deduct these payments, leaving as a new principal $412.10. [N. B. — Each new principal must be less than the preceding, otherwise there will be interest on interest.]

The amount of this new principal at the time of the 3d payment was $443.49; and as this payment, $100, was more than the accrued interest, $31.39, our 2d new principal is $343.49.

Finally, it is evident, without careful computation, that the next payment, $5, over 5 mo. from the last, did not equal the interest then accrued. So we compute the interest on this 2d new principal, $343.49, to the time of settlement, and, finding the amount, deduct the $5. This leaves the amount due on the note $351.37.

3. $504. CLEVELAND, O., June 10, 1869.

On demand, for value received, I promise to pay Zenas White, or order, five hundred and four dollars ($504), with interest at 6% per annum. J. A. KING.

On this note were the following indorsements: —

Jan. 25, 1870, $84; May 15, 1870, $100;
Feb. 20, 1871, $200.

What was due July 5, 1871?

4. $450. LOUISVILLE, KY., Jan. 1, 1865.

Two years after date, for value received, I promise to pay to the order of James Jones four hundred and fifty dollars ($450), with interest at 8% per annum.

RANDALL WRIGHT.

On the back of this note were the following indorsements: —

March 16, 1865, $75; Jan. 1, 1866, $100;
April 4, 1866, $200.

What was due on the note Jan. 1, 1867?

5. Date of note, March 11, 1870; face, $58.50; rate per cent, 10. Payments: June 5, 1871, $12; Nov. 23, 1873, $6; *Aug.* 7, 1874, $5; Dec. 18, 1874, $20; May 10, 1876, $5. *How much* remained due July 1, 1876?

6. On a note of $400, at 7%, there was paid $100 annually for 3 years. How much remained due 3 yr. 4 mo. from the date of the note?

MERCHANT'S RULE.

362. It is a common practice with business-men to treat obligations maturing and settled in a year or less, and upon which payments have been made, according to the following

Rule. — *Find the amount of the principal from the date of the note to the time of settlement, find the amount of each payment from the time it was made to the time of settlement, and subtract their sum from the first result.*

1. $250.60. ANN ARBOR, July 7, 1876.

For value received, I promise to pay Stephen Beckwith, or order, two hundred and fifty and $\frac{60}{100}$ dollars, April 15, 1877, with interest at 7%. EDWARD SNOW.

Indorsed Sept. 20, 1876, $80.00.
" Jan. 1, 1877, $50.00.
" Mar. 13, 1877, $50.00.

What was the amount due April 15, 1877?

Reckoning the time by the Banker's Method (i.e., using exact time and the common interest tables, but without grace), we have, —

Amount of note April 15, 1877,	$264.34
Amount of 1st payment, from Sept. 20 to April 15,	$83.22
" 2d " " Jan. 1 "	51.01
" 3d " " March 13 "	50.32
Total amount of payments,	$184.55
Balance due April 15, 1877,	$79.79

2. Date of note, Aug. 23, 1874; principal, $420; rate, 10%; nominal maturity, May 1, 1875. Indorsements: $100

Oct. 15, 1874; $200 Jan. 1, 1875. What was due May 1, 1875, reckoning the time by the Banker's Method?

3. $500. RICHMOND, Jan. 1, 1875.

Ninety days after date, for value received, I promise to pay to the order of Frank H. Ransom five hundred dollars, with interest at 6%. JOHN M. SABIN.

Indorsements: Jan. 20, $100; Feb. 10, $50; Feb. 25, $100; March 1, $150.

What was due at maturity (353), Banker's Method?

4. $400. BUFFALO, Jan. 1, 1874.

One year after date, for value received, I promise to pay N. Stacy, or order, four hundred dollars, with interest at 7%. M. M. DEYOUNG.

Indorsements: March 16, 1874, $200; July 1, 1874, $100.

What was due at maturity?

5. $700. DANBURY, Feb. 17, 1874.

Six months after date, for value received, I promise to pay John Gordon, or bearer, seven hundred dollars, with interest at 6%.

Received on the above, May 10, 1874, $350.
" " " June 25, 1874, $200.

When did the note mature, and what was due?

VERMONT RULE.[1]

Partial Payments on Notes with "Annual Interest."

363. When partial payments are made on notes, with *Interest* "payable annually," at other times than those at which the annual interest falls due, the method usually adopted is as follows: —

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[1] The *Old Vermont Rule*, which was in quite general use in the country half a century ago, was the same as the *Merchant's Rule*, without limitation to "notes running a year or less." This rule, now known as the *Vermont Rule*, because first adopted by the courts of Vermont, has since been adopted by several other States, *and is the one* commonly used in computing interest on notes when the interest is *payable annually*, and partial payments have been made.

Find the interest on the note for 1 year; and find also the amount of the payments made during the year, from the times they were severally made to the end of the year.

If the payments amount to more than the interest due, take their amount from the amount of the note, and make the remainder a new principal.

But, if the amount of the payments does not equal the interest due, the principal remains unchanged; and the amount of the payments is taken from the interest, the remainder being treated as deferred interest.

Proceed in this manner with each year till the time of settlement, the last period being that from the time the last annual interest fell due to the time of settlement.

(*a*) The times at which interest falls due, and to which interest on payments is reckoned, and at which the amounts of the payments are applied, are called *Rests*. Courts have allowed these rests to be made at Jan. 1 on such notes, instead of at the time at which annual interest fell due. In some cases, banks have been allowed to make these *Rests* quarterly.

(*b*) In NEW HAMPSHIRE, if a payment made on a note bearing interest payable annually is less than the interest then due, it is carried forward, and added to the next payment *without interest*, and so on till the sum does exceed the interest, or to the time of settlement, when it is deducted; but when payments are made expressly on account of interest accruing, but not then due, they are applied when the interest falls due, *without interest* on so much of such payments as is necessary to cover the interest accruing.

Ex. 1. — On a 10% note for $600, with interest payable annually, and dated June 12, 1873, there were the following indorsements: —

June 12, 1874, $60;	Dec. 5, 1874, $100;
April 10, 1875, $60;	Nov. 4, 1876, $30.

What remained due Jan. 5, 1877, by the Vermont Rule?

Due June 12, 1874	$660.00
Paid June 12, 1874	60.00
Balance due June 12, 1874	$600.00
Interest for 1 year	60.00
Amount due June 12, 1875	$660.00
Amount of two payments made during this year; i.e., $100 for 6 mo. 7 da. and $50 for 2 mo. 2 da. ($105.19 + $50.86)	$156.05
Balance due June 12, 1875	$503.95
Interest on this for 1 year	50.395
Amount on interest from June 12, 1876, to settlement Jan. 5, 1877, 6 mo. 23 da.	$554.345
Interest on above for 6 mo. 23 da.	31.258
Amount due Jan. 5, 1877	$585.603
Less amount of $30 payment for 2 mo. 2 da.	30.51
Balance due on settlement	$555.19

2. On a note for $1000, bearing 8% interest payable annually, and dated July 27, 1873, there were the following indorsements: Jan. 1, 1874, $50; Sept. 19, 1875, $150; July 27, 1876, $200; Feb. 3, 1877, $250. What was due Sept. 1, 1877? (Reckon calendar months.)

$500. CONCORD, N.H., June 7, 1873.

3. On demand, for value received, I promise to pay Enos Ames, or order, five hundred dollars, with interest at 6%, payable annually. AMOS WHITE.

Indorsements: Feb. 10, 1874, $15; Aug. 15, 1874, $25; May 17, 1875, $150; Jan. 13, 1876, $20. What was due, reckoning calendar months, Oct. 18, 1876? What, if the payments were made "on interest accruing"?

USURY LAWS.

364. Usury Laws are laws regulating the rate of interest, or manner of reckoning it.

365. Legal Interest. — *Legal Interest* is the rate per cent established by law as that which is to be implied in an interest-bearing obligation in which the rate is not specified.

In Louisiana the legal rate is	5%.
In the N. E. States (except Conn.), N. C., Penn., Del., Md., Va., W. Va., Tenn., Ky., O., Mc., Miss., Ark., Io., Ill., Ind., the Dist. of Columbia, and debts due the United States .	6%.
In N. Y., Conn., N. J., S. C., Ga., Mich., Minn., Kan., and Wis.	7%.
In Ala., Fla., and Tex.	8%.
In Col., Neb., Nev., Ore., Cal., and Washington Territory .	10%.
In England and France	5%.
In Canada, Nova Scotia, and Ireland	6%.

366. **Usury** is a higher rate of interest than is lawful. Most of the States allow interest above the legal rate, when it is agreed upon between the parties and specified in the contract. Thus O. and La. and Mich. allow any rate up to 8%; Ill., Io., Miss., Wis., Mo., and Tenn., up to 10%; Minn. and Tex., up to 12%; Neb., 15%; Kan., 20%; Mass., R. I., Fla., Ark., Cal., Nev., Col., any rate agreed upon.

Of course, in those States where parties are prohibited from making a contract for more than a certain rate, *Compound Interest* is illegal, and cannot be collected at law.

367. In computing interest or discount, "a year" is a calendar year, and "a month" a calendar month. To this there are no exceptions in the States. Also, when years, months, and days are mentioned in the contract, the days are reckoned as 30ths of a month. But, in transactions with the General Government, the month unit is dropped, and the time is reckoned in years and days, the days being called 365ths of a year. In New York, when time is specified in days, the days are to be reckoned as 365ths of a year.

368. Notes falling due on Sunday, or on a legal holiday, are in most of the States required to be paid on the preceding day. In Connecticut, if the day of maturity is a legal holiday falling on Sunday, the note is due on Monday. In Maine and Nebraska, if the day of maturity is a legal holiday falling on Monday, the note is payable on Tuesday; and in New York a note maturing on a legal holiday, or Monday observed as such holiday, is payable the following day.

369. In the following States, simple interest can be collected on unpaid annual interest; viz., Michigan (same rate as borne by the note), Ohio, Wisconsin, Vermont, New Hampshire, Iowa (6%). In Pennsylvania, Georgia, Illinois, and Indiana, by special contract (only). In Massachusetts such annual interest can be sued for when due; but no interest can be collected on it.

370. In Pennsylvania a note for 30 da. is discounted at bank for 34 da.; one for 60 da., for 64 da.; one for 90 da., for 94 da. This practice comes from counting both the day on which the note is drawn and the day on which it falls due. In the ordinary practice, only one of these is counted.

SECTION IV.

DISCOUNT.

371. **Discount** is a general term used by business-men to signify any deduction made from a *nominal* price or value. There are *three* principal uses of the term; as in what is called *Commercial or Trade Discount*, *Bank Discount*, and *True Discount*.

COMMERCIAL OR TRADE DISCOUNT.

Ex. 1. — I asked a bookseller the price of a certain book. He answered, "The list price is $15; but I can allow you a *discount* of 30%." What did he ask me for the book?

By a "*discount* of 30%," he meant 30% less than $15. 30% of $15 is $4.50. Hence he proposed to sell me the book for $15 — $4.50, or $10.50. The $4.50 may be called the *Commercial* or *Trade Discount*.

A slightly modified form of *Trade Discount* is illustrated by the following example: —

2. A Western shoe-merchant buys of a Boston dealer, on 60 days' time, the following bill: the understanding being. that, if payment is made in 30 da., he shall have "2% off;" and if in 10 da., 3%: —

2 cases boots, $30 . . .	$60.00
3 cases " $96 . . .	$288.00
½ case shoes, $90 . . .	$45.00

What amount will pay the bill in 10 da.? What in 30 da.?

The purpose of this arrangement is to make it for the interest of the purchaser to pay as soon as possible. Thus, in this case, if he does not pay till the expiration of the 60 da., he must pay the full amount of the bill, $393.00; but, if he pays within 10 days, he gets a discount of 3%, or $11.79, having only $381.21 to pay. If he pays any time between 10 and 30 days, or on the 30th, he gets 2% off (that is, $7.86), and has to pay $385.14.

372. Commercial or Trade Discount is a deduction from the *nominal* price, or value, of an article, or from the amount of a bill of purchase for payment before it falls due.

[For the solution of the following, the principles of Simple Percentage are adequate.]

3. Having bought a bill of goods amounting to $250 on 90 days' credit, the tradesman says to me, "For cash I could discount you 10% on this bill." What amount of money would pay the bill *now*?

4. A certain article is marked to sell at 25% advance on cost; and the dealer gives me 10% off from retail price, and I pay $6.75 for it. What was the cost?

5. I buy of A. T. Stewart & Co., on 4 mo. time, a bill of goods amounting to $500; the rule of the house being to allow 6% off if payment is made in 10 da., and 5% if made in 30 da. What amount will pay the bill in 10 da.? What in 30 da.?

6. I paid $2.20 for a book on which the bookseller allowed me 20% discount from the retail price. What was the retail price?

7. What is saved by paying a 60 da. bill for $1200, $\frac{1}{2}$ in 10 da. at 5% discount, and $\frac{1}{2}$ in 20 da. at 4% discount? (Interest on the money not considered.)

8. I buy goods for $350 on 30 da., and for $500 on 60 da., and pay the former in 10 da. with $3\frac{1}{2}$% discount, and the latter in 20 da. with 5% discount. How much better is this than 10% per annum for my money for the time I anticipate the payments? (Exact interest.)

9. A dry-goods merchant, finding a piece of cloth which cost him \$3.75 per yard somewhat damaged, offered it for sale at 10% discount. What did he ask per yard for it?

10. A merchant sold some damaged cloth at \$3.37½ per yard, which was at a discount of 10% from the cost. What was the cost per yard?

11. A merchant sold cloth, which cost him \$3.75 per yard, at \$3.37½. What per cent did he discount?

12. What is the cash value of a bill amounting to \$3750 at 10% discount, and 2½% off for cash?

By this is meant, that, for the usual time which the house allows credit, they will sell the purchaser a bill of goods of \$3750, reckoned at the regular rates, for 10% off; but for cash down they will deduct 2½% from this.

13. What is the cash value of a bill of goods amounting to \$2157.25 at 15% discount, and 3% off for cash?

BANK DISCOUNT.

Ex. 1. — John Smith desiring to borrow some money *at a Bank*, they tell him that they can "*accommodate*" him, and are "*discounting*" at 8%. They then furnish him a blank note, which, when filled out and signed, reads as follows: —

\$200. ANN ARBOR, May 5, 1879.

Sixty days after date I promise to pay to the order of James F. Royce two hundred dollars at the First National Bank, value received. JOHN SMITH.

Mr. Smith's friend, Mr. Royce, then writes his name on the back of the note, and becomes his *Indorser*. The note is then taken to the bank by Mr. Smith; and they take it, and pay him \$200, less the interest on \$200 for 63 days at 8%; i.e., \$200 — \$2.80, or \$197.20. The \$197.20 is called the *Proceeds*, or *Avails;* and the \$2.80 is the "*Discount.*"

373. Bank Discount is interest paid in advance, and for 3 days more than the nominal time. These 3 days are called *Days of Grace.* (See **352.**)

The above note "*Matures*" (i.e., falls due) July 7, 1879, 63 days after date. If Mr. Smith does not pay it before the close of business-hours on that day, the bank sends Mr. Royce a notice called a "*Protest.*" This notice states that Mr. Smith has failed to pay his note, and that the bank now holds Mr. Royce responsible for it. This makes Mr. Royce liable, and he must pay the note if Mr. Smith does not.

2. April 29, 1879, wishing to raise a little money, I find I have a good note against Mr. E. Wright for $240, dated Jan. 10, 1879, bearing 10% interest, and due 6 mo. after date. I take this to the bank, and find that they will discount it for me at 8% if I will indorse it. I put my name on the back, and hand it in. How much money do I receive?

Since Mr. Wright is not obliged to pay this note till 3 days after it is nominally due (i.e., until July 13), the bank will reckon interest on it to that date, and discount it accordingly. When it matures, the note will bring $252.20. The bank discount on this for the 75 da. from April 29 to July 13, at 8%, is $\frac{252.20 \times 8 \times 75}{360 \times 100} = \$4.20$. Hence I shall receive $248.

3. In many banks, as in those of New-York City, bank discount is reckoned as *Exact Interest* in advance. In such a bank, what is the discount on a note of $5000 for $^{60}/_{63}$ days at 10% per annum?

A note given at bank for 30 da. matures in 33, and the time is usually written $^{30}/_{33}$. So, also, $^{60}/_{63}$ da. means *nominally* due in 60 da., but *legally* in 63.

4. $175. ANN ARBOR, MICH., Feb. 23, 1879.

Sixty days after date I promise to pay to the order of Jas. F. Royce one hundred and seventy-five dollars at the Ann-Arbor Savings Bank, for value received, with ten per cent interest after due. EDWARD OLNEY.

This note, being indorsed by Mr. Royce, was discounted

at the savings bank at 10% per annum on the day of its date. What were the proceeds?

5. I have a 7% note for $500, dated Jan. 25, 1874, and nominally due Dec. 10, 1875. I get it discounted at bank for 10% Sept. 6, 1875. What are the proceeds?

6. April 1, 1879, a merchant, being in need of ready money, finds that he has the three following notes: —

(1) Mr. Brown's note for $350, bearing 7% interest, dated Feb. 6, 1879, and due 4 mo. after date;

(2) Mr. Jones's note for $300, bearing 6% interest, dated March 1, 1879, and due 2 mo. after date; and

(3) Mr. Smith's note for $150, bearing 8% interest, dated March 12, 1879, and due 3 mo. after date.

These being good notes, he indorses them (i.e., puts his name on the back of each), and, taking them to the bank, gets them discounted at 8%. How much ready money does he raise?

In finding the amount of a note in such a case, reckon *calendar* months, and any excess of days (with grace) as 30ths of a month. In computing the discount, reckon the exact number of days (with grace), and call them 360ths of a year. This would be the common practice of banks.

7. June 10, 1879, for value received, I promise to pay Enos White, or order, $320.

DETROIT, Jan. 5, 1879. JOHN EZRA.

By indorsing this note, Mr. White got it discounted at bank March 10, 1879, at 7%. What were the proceeds?

Such a note does not draw interest.

374. A **Negotiable** note is a note which the holder may sell to another person, who shall have legal power to collect it. The words "*or bearer*" make a note negotiable without indorsement. "*Or order*" requires indorsement. If no such phrase is contained, the note is not negotiable, either *with or* without indorsement.

8. $750. BOSTON, June 16, 1878.

Nine months after date, for value received, I promise to pay Mary Smith, or order, seven hundred fifty dollars, with interest at 6 per cent.

JOHN E. HOWE.

What is the discount, at 6%, Oct. 24, 1878?

How would this note be made negotiable?

9. $375. CHICAGO, ILL., Dec. 20, 1876.

Sixty days after date, for value received, I promise to pay E. D. Bronson, or bearer, three hundred seventy-five dollars, with interest at 10%, at the First National Bank, Chicago, Ill. S. HOWARD BLACKWELL.

This note was discounted Jan. 23, 1877, at 10%. How was this note made negotiable? What were the proceeds?

Technically and legally, such a note is negotiable without indorsement; but it is the custom of *banks* to require indorsement, the same as when drawn payable to "order."

10. A note of $1400, dated July 19, 1877, due May 1, 1878, with interest at 6%, and discounted Jan. 17, 1878, at 10%. What were the proceeds? (No grace.)

11. A note of $2400, dated Oct. 16, 1877, due Jan. 1, 1879, with interest at 8%, and discounted July 26, 1878, at 10%. What were the proceeds? (No grace.)

12. CLEVELAND, Aug. 7, 1878.

Four months after date, for value received, I promise to pay Mr. Elisha Jones five hundred dollars, with interest at 8%. JOHN GORTON.

Mr. Jones, having written his name on the back of this note, sells it to Peter Dull, who gets it discounted at a bank Oct. 10, 1878, at 10%. What are the avails?

13. A note for $6000 was made May 10, 1868, payable in six months with interest at 9%, and discounted at a bank in Michigan, Oct. 3, 1868, at 7%. What were the proceeds?

14. What is the bank discount on a note for \$250 (*Exact Int.*) at 7% for $^{60}/_{63}$ da.? For $^{30}/_{33}$ da.? At 10% for $^{90}/_{93}$ da.? For $^{30}/_{33}$ da.?

EXPEDITIOUS METHODS OF COMPUTING BANK DISCOUNT FOR 33, 63, AND 93 DAYS.

[As most Bank Paper is made for one of these three times, the following simple and elegant methods are worth knowing.]

375. ***General Method. — For 12 Per Cent.***

Calling 360 da. a year, to obtain the interest at 12% on any principal

For 33 da., take 11-1000ths of the principal.
For 63 da., take 21-1000ths of the principal.
For 93 da., take 31-1000ths of the principal.

DEMONSTRATION. — For 12% for 33 da., letting P represent the principal, we have

$$\frac{\cancel{33}^{11} \times \cancel{12}^{1} \times P}{\cancel{360}^{30} \times 100} = \frac{11}{1000} \text{ of P.}$$

$$\cancel{120}\ 10$$

The others are demonstrated in the same manner.

Observe that to take $\frac{11}{1000}$ is to multiply by 11, and remove the decimal point three places to the left.

To multiply by 11, write the principal under itself, removing it one place to the left, and add; to multiply by 21, write 2 times the principal in the same way; to multiply by 31, write 3 times the principal in the same way.

Ex. 1. — Find the interest on \$5872 at 12% for 33 da., 63 da., 93 da.

\$5 872	\$5 872	\$5 872
58 72	117 44	176 16
\$64.59 Int. for 33 da.	\$123.31 Int. for 63 da.	\$182.03 Int. for 93 da.

376. ***For other Rates Per Cent than 12.***
First find 12% as above. Then,

For 6%, take $\frac{1}{2}$ of 12%.
For 7%, add $\frac{1}{6}$ to 6%.
For 8%, deduct $\frac{1}{3}$ from 12%.
For 9%, deduct $\frac{1}{4}$ from 12%.
For 10%, deduct $\frac{1}{6}$ from 12%.

NOTE. — When the principal is a round number of hundreds of dollars, 12% can be told at a glance. Thus 12% on $300 for 33 da. is $3.30; on $500, $5.50; on $700, $7.70, etc. Again: 12% on $300 for 63 da. is $6.30; on $800, $16.80; on $700, $14.70, etc. For 93 da. 12% on $200 is $6.20; on $100 is $3.10; on $400, $12.40. Thus it will be seen that for 33 da. the dollars in the discount are the hundreds of dollars in the principal; for 63 da., twice the hundreds; for 93 da., three times the hundreds, — the cents in each case being the principal with the right-hand 0 dropped.

2. Find the bank discount on $600 at 6% for 33 da.; 63 da.; 93 da.

$6.60	$12.60	$18.60
$3.30 for 33 da.	$6.30 for 63 da.	$9.30 for 93 da.

3. Find the bank discount on $200 at 8% for 33 da.; 63 da.; 93 da.

$2.20	$4.20	$6.20
.73	1.40	2.07
$1.47 for 33 da.	$2.80 for 63 da.	$4.13 for 93 da.

4. Tell mentally the bank discount on $200 for 33 da. at 12%; 6%; 7%; 8%; 9%; 10%. Also for 63 da. For 93 da.

5. Compute as above the bank discount on $756.80 for 33 da. at 12%. At 6%; 7%; 8%; 9%; 10%. Also for 63 da. and for 93 da.

To find the Face of a Note to be made at Bank, in order to obtain a Given Sum as Proceeds.

Ex. 1. — I wish to obtain \$500 at bank for $^{60}/_{63}$, and they are discounting at 10%. For what amount must I draw my note?

\$1 face of note gives \$1.00 — \$.0175 = \$0.9825 proceeds, since the interest on \$1 for $^{60}/_{63}$ at 10% is \$0.0175. Hence, to obtain \$500, I must make my note for $\frac{\$500}{.9825} = \$508.91$.

PROOF. — If I make a note for \$508.91 for $^{60}/_{63}$, the bank will deduct from the face of the note the *Bank Discount*, which is the interest in advance. Now, the interest for \$508.91 for $^{60}/_{63}$ da. at 10% is $\$508.78 \times \frac{1}{10} \times \frac{63}{360} = \$8.81$. Hence the proceeds of such a note are \$500.

377. Rule. — *Find the interest of \$1 for the given rate and time (including 3 da. grace), and, deducting this interest from \$1, divide the sum desired by the remainder. The quotient is the face of the note.*

[Reckon 360 da. a year.]

2. For what must I draw my note in order to obtain \$50 at bank for $^{30}/_{33}$ da., when they are discounting at 8%? Give proof.

3. For what must I draw my note at bank for $^{90}/_{93}$ da. in order to obtain \$1000, when they are discounting at 7%? Give proof.

4. What is the bank discount on a note for $^{45}/_{48}$ da., which yields \$2500 proceeds, at 9%?

5. In order to obtain \$350 at bank for $^{30}/_{33}$ da., what must be the face of my note, discount being at 7%? What, to get \$150 for $^{90}/_{93}$ da. at 10%? To get \$750 for $^{60}/_{63}$ at 5%?

6. I owe a bill for flour and feed amounting to \$73.25, and give my note for 90 da. How must I draw it to cover the discount at 8%?

7. Sold a horse for $250, a carriage for $175, and a set of harness for $120, and took the purchaser's note for 90 da., so as to cover the discount at 6%. What was the face of the note?

8. Bought a bill of goods amounting to $2500 on 3 months' credit without interest. What should I be required to pay down, money being worth 10%?

9. A merchant buys a bill of goods, which he can have at $^{60}/_{63}$ da. credit, for $2850, or for $2800 cash. He can borrow at bank for 8%. Would it be better to do so? What would be the difference?

[Practically, banks do not use the above process. Were I in want of $300 for 60 da., and a bank was willing to "accommodate" me at 8%, the cashier would see from his tables that the discount on $300 would be $4.20, and would say, "Make your note for $305." The proceeds of this would be $300.73.]

TRUE DISCOUNT.

1. I have a note due 2 years hence, which will bring me at that time $228. I wish to obtain the money on it *now*. What is it worth, the use of money being worth 7% per annum?

The supposition is that the use of $1 for a year is worth to me $0.07, and for 2 years $0.14. Hence every $1.14 of the $228 due 2 years hence is worth to me $1 *now*. $228 ÷ 1.14 = 200. Therefore the note is worth $200 *now*. The $200 is called the *Present Worth*. $228 — $200, or $28, is the *Discount*. To distinguish this from *Bank Discount*, it is usually called *True Discount*.

378. True Discount is a deduction made for the present payment of a sum of money due at some future time.

379. The Present Worth of a sum of money due at some future time is a sum which, put at interest at a rate agreed upon, will in the given time amount to the sum due.

2. I take a note for $300, bearing interest at 7%, and due 3½ years hence. What is its present worth, money being worth 10%?

SOLUTION. — The *amount* of the note at maturity will be $373.50. Now, $1 at 10% will amount to $1.35 in the given time. Hence the *Present Worth* of said note is 373.50 ÷ 1.35 = 276.66⅔, or $276.66⅔.

PROOF. — That this is just appears from the fact, that, if I retain the note, I shall get $373.50 at the expiration of 3½ years; while if I sell it for $276.66⅔, and the money is worth 10% to me, I shall realize the *amount* of $276.66⅔ at 10% for 3½ years, or $373.50.

3. I take a note for $300, bearing interest at 10%, due 3½ years hence. What is its present worth, money being worth 7%?

The *amount* due on the note at the end of the time will be $405. But, as money is now worth only 7%, $1 in hand now will amount to $1.245 in the 3½ years. Hence the present worth is 405 ÷ 1.245.

That this note is worth *more than its face* ($300) is evident, since it is drawing a *higher rate of interest* than money is *now* worth.

380. **The Face of a Note** is commonly understood to be the principal, or that portion of the principal which is unpaid. Some, however, use the phrase as signifying what is due at the time; while others use it as signifying *the Amount of the Note at Maturity.*[1]

381. When the *True Present Worth* of a note exceeds the *face* of the note, this excess is called **Premium.**

382. The difference between the nominal present value (as the *face* of a note) and the *True Present Worth* is the **True Discount,** or **Premium,** as the case may be.

To find the Present Worth of a Sum of Money due at some Future Time.

383. **Rule.** — *Divide the sum due at the future date by the amount of $1 at the rate agreed upon for the time from which it is proposed to discount the sum till the time said sum is due. The quotient is the Present Worth.*

[1] *I find no authority in the books for any thing but the former.* — AUTHOR.

4. What is the true present worth of the following note, May 13, 1879, discounted at 8%?

$276. GRAND RAPIDS, MICH., Aug. 7. 1877.

For value received I promise to pay Eber White, or order, Dec. 10, 1879, two hundred seventy-six dollars, with interest at 6%. PETER DULL.

5. I have a 7% note for $186.50, dated Feb. 7, 1876, and due Sept. 20, 1879. What is its true present worth July 17, 1878, discounting at 10%?

6. Jan. 14, 1878, a speculator offered me $300 for a note of $350, dated May 7, 1877, payable Oct. 21, 1879, and bearing 6% interest, money being worth 10%. Did he offer me the full value of the note?

7. I have a 10% note for $280, dated Sept. 17, 1876, and due Feb. 6, 1879. May 23, 1878, Mr. C. proposes to buy it of me, discounting at 8 %. What must he pay me?

8. Mr. C. gives me his note for $300, due 2 yr. hence, at 10%, and I sell it to Mr. B. the same day at 8% discount. What does B. pay me?

Why is this note worth more than its face?

9. Mr. C. gives me his note for $300, due 2 yr. hence, at 8%, and I sell it the same day to Mr. B. at 10% discount. What does B. pay me?

Why is this note worth less than its face?

10. Mr. C. gives me his note for $300, due 3 yr. hence, at 10% interest, which is all that money is worth. What is the present worth of the note on the day it is made? One year after its date, what is its present worth? Two and one-half years after date?

11. Mr. C. gives me his note for $300, due 3 yr. hence, without interest. What is it worth on the day it is given, money being worth 10%? What 1 yr. after date? What 2½ yr. after date? What 3 yr. after date?

12. A merchant bought goods amounting to $4200 on a credit of 6 months, without interest. What sum in ready money would discharge the debt, money being worth 8% per annum?

13. A note, $52.25, dated Sept. 12, 1872, and bearing 10% interest, will be paid Jan. 1, 1879. What is it worth July 1, 1877, money being worth 6%?

14. Having bought a bill of goods amounting to $750 on 90 days' credit, the tradesman says to me, "For cash I could discount you 8% on this bill." What amount of money would pay the bill *now?*

15. Bought goods to the amount of $840.40 on 4 mo. credit, without interest. How much money would discharge the debt at the time of receiving the goods, discounting at 8% per annum?

16. July 1, 1878, I discount at 8% a 7% note for $275, dated Oct. 15, 1877, and due Jan. 12, 1880. What is the present worth?

17. April 10, 1879, I wish to raise some money, and find I have the three following notes: —

A 6% note for $225, dated Aug. 7, 1878, due Jan. 1, 1880;

A 7% note for $156, dated Nov. 1, 1877, due May 3, 1880;

A 7% note for $200, dated Feb. 3, 1878, due Oct. 14, 1878.

A friend is willing to buy the notes at 8% discount. How much money can I raise on them?

SECTION V.

GOVERNMENT BONDS.

384. The **Bond** of a corporation is its certificate of indebtedness, signed by the proper officers, and given under the corporate seal.

Such bonds are the notes of railroad, manufacturing, or other corporations, and are usually secured by mortgage upon their property. These bonds, like other notes, are made payable at a certain time, and bear a specified rate of interest. The bonds of a corporation are usually considered a surer investment than the stocks, since *dividends* on the latter are made only when the business receipts exceed the expenditures; whereas the *interest* on the former is due at the times named, whether there be profits in the business or not, and, when the bonds are secured by mortgage, the mortgage may be foreclosed like other mortgages. Nevertheless, in some very lucrative business, the stocks may be more valuable than the bonds.

385. **Government Bonds** are certificates of indebtedness issued by the government; as by the United-States or State government, by a county, city, school-district, or other government corporation.

Such bonds are usually made payable at a certain time, and bear a specified rate of interest.

The occasions for these bonds are such as the following: When a school-district wishes to build a fine house, but does not want to increase the taxes sufficiently to pay for it in a single year, but prefers to distribute the payment over several years; and, in like manner, when a county or state is called upon to expend more money than it is deemed expedient to raise by immediate taxation. But the fruitful cause of such government indebtedness is *war*. In consequence of our late war, the indebtedness of the United-States Government ran up from $88,995,810 in 1861 to $2,639,382,572 in 1867. In like manner, England and France have accumulated enormous debts.

386. English **Consols** are government stocks. **Rentes**[1] are **French** government stocks.

[1] Pronounced "*rahnts*." In strict language, the term *Rentes* applies only to the *interest*; the principal — the debt itself — being called *Nominal Capital*.

(*a*) English *Consols* are properly but perpetual 3% *annuities*, as the principal of the debt is not presumed to be payable. The consolidated debt of England is £731,413,523. The term *Consols* is applied also to certain United-States bonds.

(*b*) The principal part of the United-States interest-bearing debt in the market April 1, 1879, was,[1]—

5s of 81	$508,440,350
6s of 81	264,321,350
4½s (1891)	250,000,000
4s (1907)	556,467,950

(*c*) "5s of 81" means government bonds, which bear 5% interest, payable quarterly on Feb. 1, May 1, Aug. 1, and Nov. 1.

(*d*) "6s of 81" are 6% bonds, interest payable semi-annually on Jan. 1 and July 1.

Both of these classes of bonds are redeemable by the government in 1881, — the former May 1, and the latter June 30. No doubt the government will be ready to redeem them then, and interest on them will cease. There are also about 18½ million 6s of 80, which mature Dec. 31, 1880.

In this statement nothing is said of the old 10-40s and 5-20s, since the interest on the last of these expires July 1, 1879, and the government has already made ample provision for redeeming them. The total interest-bearing debt of the United States, April 1, 1879, including these outstanding 10-40s and 5-20s, was $1,968,962,800. This includes the 356 millions of 10-40s and 5-20s, and also the 4% which have been sold to redeem them. Hence our interest-bearing debt, after July 1, 1879, will be about $1,600,000,000.

(*e*) The 4½s bear 4½% interest, payable quarterly March 1, June 1, Sept. 1, and Dec. 1, and are redeemable Sept. 1, 1891.

(*f*) The 4s bear 4% interest, payable quarterly Jan. 1, April 1, July 1, Oct. 1, and are redeemable July 1, 1907.

387. A Coupon is a certificate of interest attached to a bond, which, on the payment of the interest, is cut off, and delivered to the payor.

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[1] The figures given above are from the official report of treasury April, 1879; but the immense sales of 4% during April and May make the 4s in the market now (*May 20*) nearer 800 million.

Stocks and bonds are bought and sold in the market just as wheat or cotton, and the prices fluctuate according to prosperity of business, the plenty or scarcity of money, and many other circumstances.

United-States bonds are of two classes, — *Registered* and *Coupon*. Registered bonds have to be indorsed by the owner when he sells them; and the government keeps an account of such transfers, and hence can at any time tell who owns a particular bond. The owner of a *Coupon* bond may lose it, or it may be stolen; and the person who has it in possession can collect the interest, or sell the bond. But registered bonds are not liable to such contingencies. Yet the impracticability of making transfers of registered bonds in foreign countries confines transactions abroad to coupon bonds.

388. Pacific Railroad Bonds are the bonds of these corporations guaranteed by the United States. Of these there are $64,623,512 in the market, bearing 6% interest, which is paid by the United-States Government. These are not reckoned a part of the United-States debt, since the government holds the first-mortgage bonds on the entire property of the companies to secure the payment.

389. Quotations are the statements made from day to day in the *newspapers*, giving the rates at which exchange, stocks, bonds, etc., are being bought and sold in the money market.

"Interest to buyer" means that the buyer has the advantage of whatever interest has accrued at the time of purchase. Thus, if I buy a $100 bond, "6s of 81," quoted at 106½ (which means $106.50 for a $100 bond), and there has accrued $2.10 interest on that bond at the time of my purchase, the bond really costs me $106.50 — $2.10, or $104.40. But if I buy a Michigan 7% bond for $100, quoted "112, interest to seller," 4 mo. after the interest was paid (i.e., when 4 mo. interest has accrued), it costs me $112 + ⅓ of $7, or $114.33⅓, since I have to pay to the seller the accrued interest.

Government bonds are always sold "interest to buyer." Other bonds are sometimes quoted "interest to seller;" which means that they are selling at the quotation plus the accrued interest at the time of sale.

390. *Examples.*

[We give examples showing how to find the cost of bonds, and what rate per cent an investment will yield. These are the two most important problems, and are solved by principles already made familiar.]

1. What cost a $500 United-States bond, 6s of 81, at 116½?

2. What cost a $1000 Tennessee 6% bond (old), quoted 34½?

3. United-States 6s of 81 are to-day (May 20, 1879) quoted at 107. What will a $500 bond cost me?

The quotation means that $100 of bond costs $107: but on this there is accrued (May 20) interest since Jan. 1 (see **386**, *d*); i.e., for 139 da. 6% on $100 for 139 da. is $2.29. This interest I shall receive July. Hence the bond actually costs me $107 − $2.29 = $104.71, saying nothing of the discount on the $2.29 from this time to July 1. The $500 bond, therefore, will actually stand me in $523.55, though really I pay at the purchase $535.

4. United-States 5s of 81 are quoted to-day (May 20, 1879) at 103⅝. What will a $1000 bond cost me exclusive of the interest accrued? (See **386**, *c*.)

5. What will a $500 bond, Michigan 7s (war bounty loan), quoted at "112, interest to seller," cost me 2 mo. 10 da. after the interest has been paid?

At "112" the bond will cost 5 × $112 = $560; and, as the interest accrued goes to the seller, I shall have to pay 2 mo. 10 da. interest on $500 at 7%, or $6.81 in addition.

On transactions in United-States bonds reckon the exact number of days as 365ths of a year. On State bonds reckon calendar months, and days as 30ths of a month.

6. When Erie consolidated 7s are quoted "111⅞, interest to seller," 3 mo. after payment of interest, what does a $100 bond cost?

7. When District-of-Columbia 3-65s are quoted "$86\frac{7}{8}$, interest to seller," with $2\frac{1}{2}$ mo. accrued interest, what cost a $1000 bond?

3-65s means bonds that bear $3\frac{65}{100}\%$ interest; i.e., a $100 bond bears interest at 1¢ per day, all years being reckoned as having 365 da.

8. When United-States 4s are quoted at $102\frac{3}{8}$, with 1 mo. 18 da. accrued interest, what shall I pay for a $1000 bond? What is the actual cost exclusive of accrued interest?

9. When United-States $4\frac{1}{2}$s are quoted at 107, with 2 mo. 22 da. accrued interest, what is the actual cost of a $500 bond?

Interest and brokerage are always reckoned on the *face* of a bond or stock certificate.

10. I buy through a broker, who charges me $\frac{1}{8}\%$ for buying $2500 United-States 4s, quoted $102\frac{1}{4}$. What amount do I pay? What is the net cost of the bonds above accrued interest, if the purchase is made May 25? (See **386**, *f.*)

11. If I buy Bay-City 8s, quoted at 111, 6 mo. after their date, interest to seller, and pay my broker $\frac{1}{8}\%$, what do $5000 in bonds cost me?

As there is 6 mo. accrued interest, which I have to pay to the seller, I pay for $100 in bonds $111 + the interest for 6 mo. at 8% + 20¢ for brokerage.

12. What do I pay for $1000 United-States 5s of 81, April 15, the quotation being $103\frac{5}{8}$, and brokerage $\frac{1}{10}\%$? What do the bonds stand me in, exclusive of accrued interest at time of purchase?

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13. If I buy a $500 $7\frac{1}{2}\%$ county bond 9 mo. after its date at 105, interest to seller, what do I pay for the bond? What per cent does my investment yield?

390. *Examples.*

[We give examples showing how to find the cost of bonds, and what rate per cent an investment will yield. These are the two most important problems, and are solved by principles already made familiar.]

1. What cost a $500 United-States bond, 6s of 81, at 116½?

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4. United-States 5s of 81 are quoted to-day (May 20, 1879) at 103⅝. What will a $1000 bond cost me exclusive of the interest accrued? (See **386**, *c*.)

5. What will a $500 bond, Michigan 7s (war bounty loan), quoted at "112, interest to seller," cost me 2 mo. 10 da. after the interest has been paid?

At "112" the bond will cost 5 × $112 = $560; and, as the interest accrued goes to the seller, I shall have to pay 2 mo. 10 da. interest on $500 at 7%, or $6.81 in addition.

On transactions in United-States bonds reckon the exact number of days as 365ths of a year. On State bonds reckon calendar months, and days as 30ths of a month.

6. When Erie consolidated 7s are quoted "111⅞, interest to seller," 3 mo. after payment of interest, what does a $100 bond cost?

7. When District-of-Columbia 3-65s are quoted "86⅞, interest to seller," with 2½ mo. accrued interest, what cost a $1000 bond?

3-65s means bonds that bear $3\frac{65}{100}\%$ interest; i.e., a $100 bond bears interest at 1¢ per day, all years being reckoned as having 365 da.

8. When United-States 4s are quoted at 102⅜, with 1 mo. 18 da. accrued interest, what shall I pay for a $1000 bond? What is the actual cost exclusive of accrued interest?

9. When United-States 4½s are quoted at 107, with 2 mo. 22 da. accrued interest, what is the actual cost of a $500 bond?

Interest and brokerage are always reckoned on the *face* of a bond or stock certificate.

10. I buy through a broker, who charges me ⅛% for buying $2500 United-States 4s, quoted 102¼. What amount do I pay? What is the net cost of the bonds above accrued interest, if the purchase is made May 25? (See **386**, *f.*)

11. If I buy Bay-City 8s, quoted at 111, 6 mo. after their date, interest to seller, and pay my broker ⅛%, what do $5000 in bonds cost me?

As there is 6 mo. accrued interest, which I have to pay to the seller, I pay for $100 in bonds $111 + the interest for 6 mo. at 8% + 20¢ for brokerage.

12. What do I pay for $1000 United-States 5s of 81, April 15, the quotation being 103⅝, and brokerage $\frac{1}{10}\%$? What do the bonds stand me in, exclusive of accrued interest at time of purchase?

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13. If I buy a $500 7½% county bond 9 mo. after its date at 105, interest to seller, what do I pay for the bond? What per cent does my investment yield?

390. *Examples.*

[We give examples showing how to find the cost of bonds, and what rate per cent an investment will yield. These are the two most important problems, and are solved by principles already made familiar.]

1. What cost a $500 United-States bond, 6s of 81, at 116½?

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4. United-States 5s of 81 are quoted to-day (May 20, 1879) at 103⅝. What will a $1000 bond cost me exclusive of the interest accrued? (See **386**, *c*.)

5. What will a $500 bond, Michigan 7s (war bounty loan), quoted at "112, interest to seller," cost me 2 mo. 10 da. after the interest has been paid?

At "112" the bond will cost 5 × $112 = $560; and, as the interest accrued goes to the seller, I shall have to pay 2 mo. 10 da. interest on $500 at 7%, or $6.81 in addition.

On transactions in United-States bonds reckon the exact number of days as 365ths of a year. On State bonds reckon calendar months, and days as 30ths of a month.

6. When Erie consolidated 7s are quoted "111⅞, interest to seller," 3 mo. after payment of interest, what does a $100 bond cost?

7. When District-of-Columbia 3-65s are quoted "$86\frac{7}{8}$, interest to seller," with $2\frac{1}{2}$ mo. accrued interest, what cost a $1000 bond?

3-65s means bonds that bear $3\frac{65}{100}\%$ interest; i.e., a $100 bond bears interest at 1¢ per day, all years being reckoned as having 365 da.

8. When United-States 4s are quoted at $102\frac{3}{8}$, with 1 mo. 18 da. accrued interest, what shall I pay for a $1000 bond? What is the actual cost exclusive of accrued interest?

9. When United-States $4\frac{1}{2}$s are quoted at 107, with 2 mo. 22 da. accrued interest, what is the actual cost of a $500 bond?

Interest and brokerage are always reckoned on the *face* of a bond or stock certificate.

10. I buy through a broker, who charges me $\frac{1}{8}\%$ for buying $2500 United-States 4s, quoted $102\frac{1}{4}$. What amount do I pay? What is the net cost of the bonds above accrued interest, if the purchase is made May 25? (See **386**, *f.*)

11. If I buy Bay-City 8s, quoted at 111, 6 mo. after their date, interest to seller, and pay my broker $\frac{1}{8}\%$, what do $5000 in bonds cost me?

As there is 6 mo. accrued interest, which I have to pay to the seller, I pay for $100 in bonds $111 + the interest for 6 mo. at 8% + 20¢ for brokerage.

12. What do I pay for $1000 United-States 5s of 81, April 15, the quotation being $103\frac{5}{8}$, and brokerage $\frac{1}{10}\%$? What do the bonds stand me in, exclusive of accrued interest at time of purchase?

---

13. If I buy a $500 $7\frac{1}{2}\%$ county bond 9 mo. after its date at 105, interest to seller, what do I pay for the bond? What per cent does my investment yield?

I pay $553.125 for the bond, and it yields me $37½ interest per year. The question then is, What per cent of $553.125 is $37½?

14. In "The New-York Independent" Fisk & Hatch say, under date of May 1, 1879, that an investment in 6s of 81, which mature Dec. 31, 1880, and are selling at 106½, will barely pay 3% on the investment. How do they make it out?

Suppose I buy a $100 bond. I pay for it $106.50. On this I get $3 interest July 1, 1879; $3 Jan. 1, 1880; $3 July 1, 1880; and $3 Jan. 1, 1881. At this time the interest ceases, and the bond is worth exactly $100. I then have $112[1] for my investment of $106.50 for 1 yr. 8 mo.; i.e., from May 1, 1879, to Jan. 1, 1881. The interest on the $106.50 is, therefore, $112.00 − $106.50 = $5.50. Now, at 1%, $106.50 yields in 1 yr. 8 mo. $1.775. Hence $5.50 is 5.50 ÷ 1.775, or 3.1 times 1%, or 3.1% nearly.

15. What per cent on the investment do 6s at 95, interest payable annually, yield, if bought 3 mo. after the payment of the interest, interest to seller?

Compute on a $100 bond in all such questions. In this case such a bond costs $96.50. The question then is, What per cent on $96.50 is $6? In such computations the bond is supposed to run indefinitely, so that no note is taken of the fact that for the first year the investment yields its first interest only 9 mo. after it is made.

16. Considered as a permanent investment, what per cent on my money do 4% bonds, bought at 103¼, yield me?

17. Which is the better investment, — United-States 4½s bought at 104½ with 2 mo. accrued interest, or State 7s with 5 mo. accrued interest, bought at 107, interest to seller, considering each as a permanent investment, and taking no notice of the semi-annual payment of interest?

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[1] An exact analysis requires that we take account of the interest on the three payments made before Jan. 1, 1881. Compounding this each six months at 6%, and also compounding the interest on $106.50 in the same way, we get a little less than 3%.

SECTION VI.

EXCHANGE.

391. A merchant in Detroit wishes to pay a debt of $2500 in New York. He may send the money by a friend, by mail, or by express; but the most common and most convenient way is to step into a *bank* in Detroit, and, paying in his $2500 with a small percentage for their trouble, get the Detroit bank's order on a New-York bank for the $2500. This *order*, called a *Draft*, the Detroit merchant can send to his creditor in New York, who, by stepping into the New-York bank to which the order is addressed, will get his $2500.

A similar order given by a bank in this country upon a *foreign bank*, as one in London, Eng., is called a *Bill of Exchange.*

392. **Exchange** is a method of making payments in distant places by the use of *Drafts*, or *Bills of Exchange*, without the direct transmission of money.

When the exchange is between places in the same country, it is called *Inland* or *Domestic Exchange;* and when between places in foreign countries, it is called *Foreign Exchange.* Hence a *Draft* is a *Domestic Bill of Exchange.*

393. A **Draft**, or **Bill of Exchange**, is a written order for money, drawn in one place, and payable in another.

394. A **Bank** is a company authorized by law to issue paper money, receive deposits, deal in exchange, loan money, or buy and sell coin, bonds, stocks, etc.

Some banks make it their chief business to loan money, others to deal in exchange, others to receive deposits: while comparatively few are banks of issue; that is, issue paper currency.

2. I take a note for $300, bearing interest at 7%, and due $3\frac{1}{2}$ years hence. What is its present worth, money being worth 10%?

SOLUTION. — The *amount* of the note at maturity will be $373.50. Now, $1 at 10% will amount to $1.35 in the given time. Hence the *Present Worth* of said note is $373.50 \div 1.35 = 276.66\frac{2}{3}$, or $276.66\frac{2}{3}$.

PROOF. — That this is just appears from the fact, that, if I retain the note, I shall get $373.50 at the expiration of $3\frac{1}{2}$ years; while if I sell it for $276.66\frac{2}{3}$, and the money is worth 10% to me, I shall realize the *amount* of $276.66\frac{2}{3}$ at 10% for $3\frac{1}{2}$ years, or $373.50.

3. I take a note for $300, bearing interest at 10%, due $3\frac{1}{2}$ years hence. What is its present worth, money being worth 7%?

The *amount* due on the note at the end of the time will be $405. But, as money is now worth only 7%, $1 in hand now will amount to $1.245 in the $3\frac{1}{2}$ years. Hence the present worth is $405 \div 1.245$.

That this note is worth *more than its face* ($300) is evident, since it is drawing a *higher rate of interest* than money is *now* worth.

380. **The Face of a Note** is commonly understood to be the principal, or that portion of the principal which is unpaid. Some, however, use the phrase as signifying what is due at the time; while others use it as signifying *the Amount of the Note at Maturity*.[1]

381. When the *True Present Worth* of a note exceeds the *face* of the note, this excess is called **Premium.**

382. The difference between the nominal present value (as the *face* of a note) and the *True Present Worth* is the **True Discount**, or **Premium**, as the case may be.

To find the Present Worth of a Sum of Money due at some Future Time.

383. Rule. — *Divide the sum due at the future date by the amount of $1 at the rate agreed upon for the time from which it is proposed to discount the sum till the time said sum is due. The quotient is the Present Worth.*

[1] *I find no authority in the books for any thing but the former.* — AUTHOR.

4. What is the true present worth of the following note, May 13, 1879, discounted at 8%?

$276. GRAND RAPIDS, MICH., Aug. 7. 1877.

For value received I promise to pay Eber White, or order, Dec. 10, 1879, two hundred seventy-six dollars, with interest at 6%. PETER DULL.

5. I have a 7% note for $186.50, dated Feb. 7, 1876, and due Sept. 20, 1879. What is its true present worth July 17, 1878, discounting at 10%?

6. Jan. 14, 1878, a speculator offered me $300 for a note of $350, dated May 7, 1877, payable Oct. 21, 1879, and bearing 6% interest, money being worth 10%. Did he offer me the full value of the note?

7. I have a 10% note for $280, dated Sept. 17, 1876, and due Feb. 6, 1879. May 23, 1878, Mr. C. proposes to buy it of me, discounting at 8 %. What must he pay me?

8. Mr. C. gives me his note for $300, due 2 yr. hence, at 10%, and I sell it to Mr. B. the same day at 8% discount. What does B. pay me?

Why is this note worth more than its face?

9. Mr. C. gives me his note for $300, due 2 yr. hence, at 8%, and I sell it the same day to Mr. B. at 10% discount. What does B. pay me?

Why is this note worth less than its face?

10. Mr. C. gives me his note for $300, due 3 yr. hence, at 10% interest, which is all that money is worth. What is the present worth of the note on the day it is made? One year after its date, what is its present worth? Two and one-half years after date?

11. Mr. C. gives me his note for $300, due 3 yr. hence, without interest. What is it worth on the day it is given, money being worth 10%? What 1 yr. after date? What $2\frac{1}{2}$ yr. after date? What 3 yr. after date?

The thing required is *principal*, etc.

ANALYSIS. $1 principal yields $0.135 interest in 2 yr. 3 mo. at 6%. Now, to yield $77.76 requires as many dollars principal as $0.135 is contained times in $77.76, which is 576. Hence the principal is $576.

5. Amount $102.81 at 10% for 3 yr. 9 mo. 18 da. What is the principal?

ANALYSIS. — $1 principal at 10% for $3\frac{1}{2}$ years yields $1.38 amount. Now, to yield $102.81 requires as many dollars principal as $1.38 is contained times in $102.81. $102.81 \div 1.38 = 74.50$. Hence the principal is $74.50. [This is the ordinary problem in common discount. See **383**.]

6. Amount $102.81, on $74.50 at 10%. What is the time?

The interest is $102.81 — $74.50 = $28.31, the *given effect*. The thing required is *time*. The interest on the principal ($74.50) for 1 yr. at 10% is the effect produced by 1 of this kind.

7. Amount $102.81, on $74.50 for 3 yr. 9 mo. 18 da. What is the rate per cent? Same as Ex. 9.

8. At what per cent will $75 yield $28.125 in 6 yr. 3 mo.? At what % will it yield $15.30 in 2 yr.?

9. How long does it take $750 to amount to $942 at 6%? How long at 5%? At 3%?

10. What principal yields $150 at 4% in 7 yr. 2 mo. 15 da.? In 3 yr.? In $5\frac{1}{2}$ yr.?

11. In what time will $120 yield $16.56 interest at 6%?

12. If $584 in 2 yr. 8 mo. 7 da. yield 94.12\frac{1}{2}$, what is the %?

13. I wish to obtain $150 at bank for $^{60}/_{63}$ at 10%. For what amount must I make my note?

The *thing inquired about* is the *Face of my Note*. Now, as the interest of $1 for $^{60}/_{63}$ da. at 10% is $.01726, $1 face of note will yield $1 — $.01726, or $.98274. The *effect to be produced* is $150. Hence $150 \div .98274 = $152.63, the face of the note.

14. For what must I make my note in order to get at bank

$350 for $^{30}/_{33}$ da. when they are discounting at 8%? What to obtain the same sum at the same rate for $^{60}/_{63}$? For $^{90}/_{93}$?

15. I receive $495.50 for my note at $^{30}/_{33}$ da. at bank when they are discounting at 10%. What is the face of the note?

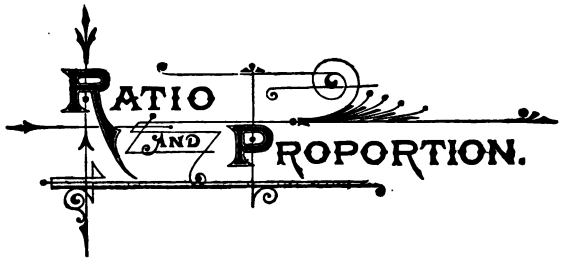
16. I receive at bank $122.90 for my note of $125 at $^{30}/_{33}$ da. What is the rate per cent of discount?

17. My annual income is $1500 from stocks, which yield 7%. What amount of stocks do I hold?

18. I have 4% bonds, which yield me $500 quarterly. What amount of bonds do I hold?

19. What amount of United-States 5s of 81 must a man have to yield an income of $400 per quarter?

20. What amount of United-States 4s must I have to yield me $600 quarterly?



CHAPTER VI.

RATIO.

402. Ratio is the quotient of one number divided by another.

Thus the ratio of 12 to 4 is $12 \div 4$, or 3. The ratio of 5 to 7 is $5 \div 7$, or $\frac{5}{7}$.

NOTE. — If the numbers are concrete, they must be of the same kind, since we cannot divide one concrete number by another of a different kind. Thus the ratio of \$10 to \$5 is 2; but to ask, "What is the ratio of \$10 to 5 miles?" is absurd.

403. The first number named is called the **Antecedent**, and the second the **Consequent**. The two together constitute the *Terms* of the ratio, or a *Couplet*.

404. The ratio between two numbers is indicated by writing the antecedent before the consequent, and the colon (:) between them; or by writing the *antecedent* as the *numerator* of a fraction, and the *consequent* as the *denominator*.

Thus 8 : 4 is read, "The ratio of 8 to 4;" so also $\frac{8}{4}$ may be read, "The ratio of 8 to 4," — both forms meaning exactly the same thing.

405. The term *Ratio* is also applied to such forms as 6 : 2, $\frac{6}{2}$, etc.; that is, to the indicated operation of division, the sign : being an equivalent of $\div$.

Thus we speak of the ratio 6 : 2 (not the ratio *of* 6 : 2), the ratio $\frac{4}{5}$; reading, "The ratio 6 to 2," "The ratio 4 to 5." The ratio *of* 6 to 2 is 3, or the *value* of the ratio 6 : 2 is 3. So the ratio *of* 4 to 5 is $\frac{4}{5}$ (4-fifths), or the *value* of the ratio $\frac{4}{5}$ (read "4 to 5") is 4-fifths.[1]

1. What is the ratio of 15 to 3? 8 : 2? 9 : 3? 10 : 2? 5 : 7? 4 : 8? 1 : 3? 3 : 1? 7 : 11? 11 : 7?

2. Which is greater, 12 : 3 or 8 : 4? 6 : 3 or 9 : 3? 5 : 6 or 7 : 8? 2 : 3 or 5 : 4? 10 : 5 or 6 : 3? 5 : 8 or 15 : 24? $\frac{4}{8}$ or $\frac{3}{6}$? $\frac{7}{11}$ or $\frac{3}{5}$?

3. Mention several ratios which are each equal to 15 : 3. Several which are equal to $\frac{8}{8}$. To $\frac{2}{3}$. To 3 : 5.

Principle.

406. *A ratio has all the properties of a common fraction with the antecedent for its numerator, and the consequent for its denominator.*

4. What effect does it produce on a ratio to *multiply* the antecedent by 2? by 3? by any number? Try it on 2 : 4. What effect to *divide* the antecedent by any number? Try it.

5. What effect does it produce on a ratio to multiply its consequent? To divide its consequent? Try it.

6. How do you compare two common fractions to ascertain which is the greater? (p. 132.) How then do you compare two ratios?

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7. If 24 is the antecedent and 4 the ratio, what is the consequent?

Having the antecedent and ratio given, how do you find the consequent?

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[1] This double use of the word "ratio" has given no little trouble to students. That the word is habitually used by mathematicians in both of these ways no one at all conversant with mathematical writing can doubt. Thus when we ask, "What is the ratio of 12 to 4?" all answer "3;" and all with equal unanimity speak of the ratio a : b, — "The ratio *a* to *b*."

8. If 3 is the consequent and 7 the ratio, what is the antecedent? If 45 is the consequent and the ratio $\frac{1}{9}$?

9. If 7 is the antecedent, what is the consequent when the ratio is $\frac{1}{7}$? When it is $\frac{2}{3}$? When it is 6?

10. If 28 is the antecedent, what is the consequent when the ratio is 7? When it is 4? When it is 14? When it is $\frac{2}{3}$? When it is $\frac{4}{5}$?

11. Antecedent 10, ratio 2, what is the consequent? Antecedent 27, ratio 9? 3? $\frac{1}{2}$?

PROPORTION.

407. Proportion is an equality of ratios, the terms of the ratios being expressed. The equality is indicated by the ordinary sign of equality (=), or by the double colon (::).

Thus 8 : 4 = 12 : 6, or 8 : 4 :: 12 : 6, is a proportion. It is read, "8 is to 4 as 12 is to 6." The expression $\frac{8}{4} = \frac{12}{6}$ may be read in the same way, and means the same thing.

408. Two ratios at least are required for proportion: hence we have two antecedents and two consequents. Of four terms which constitute a proportion, the 1st and 4th are called **Extremes**, and the 2d and 3d **Means**.

1. Is 15 : 3 :: 10 : 2 a true proportion? What is the ratio of 15 to 3? What of 10 to 2? Are they equal?

2. Show which of the following are true proportions: —

1. 20 : 5 :: 8 : 2	7. 10 : 7 :: 20 : 14
2. 2 : 12 :: 5 : 30	8. 3 : 7 :: 12 : 26
3. 5 : 35 :: 8 : 64	9. 13 : 27 :: 117 : 243
4. 2 : 3 :: 14 : 21	10. $2\frac{1}{2}$: 5 :: $3\frac{1}{3}$: $6\frac{2}{3}$
5. 7 : 11 :: 35 : 55	11. 1.05 : 8.4 :: 1 : 8
6. 8 : 3 :: 16 : 9	12. .05 : 7 :: .3 : 42

3. If the first 3 terms of a proportion are 18 : 6 :: 21, what is the 4th term?

The ratio of 18 to 6 is 3: hence the 4th term must be $\frac{1}{3}$ of 21, so that the ratios may be equal. Is 18 : 6 :: 21 : 7 a true proportion? Why?

4. Find the 4th term of 7 : 3 :: 5 : —.

What is the ratio of 7 : 3? Then, if 5 is the antecedent and $\frac{7}{3}$ the ratio, what is the consequent? Is 7 : 3 :: 5 : $2\frac{1}{3}$ a true proportion? Why?

5. Find the lacking term of 12 : — :: 8 : 6.

Which ratio is given? What is the ratio of 8 : 6? If 12 is the antecedent and $\frac{4}{3}$ the ratio, what is the consequent?

6. Find the lacking term of 13 : 7 :: — : 11.

We have the ratio of 13 : 7, $1\frac{6}{7}$. Hence the lacking term is $11 \times 1\frac{6}{7}$, or $1\frac{6}{7} = 20\frac{3}{7}$.

7. Find the lacking term of — : 43 :: 5 : 17.

The given ratio is $\frac{5}{17}$. Hence we have $43 \times \frac{5}{17} = \frac{215}{17} = 12\frac{11}{17}$. Is $12\frac{11}{17}$: 43 :: 5 : 17 true?

Principle.

409. *The product of the means of a proportion is equal to the product of the extremes.*

This is evident, since the 1st *mean* is the 1st *extreme divided* by the ratio, and the 2d *mean* is the 2d *extreme multiplied* by the ratio. Hence the product of the means is $\frac{\text{1st } Extreme}{Ratio} \times \text{2d } Extreme \times Ratio.$ In this the ratio cancels, and leaves the product of the extremes.

8. Find by means of this principle the lacking term in 13 : 5 :: 12 : —.

The two means being given, we know their product, 60; but this is also the product of the extremes. Now, if 60 is the product of the extremes and 13 is one of the extremes, the other is $\frac{60}{13}$, or $4\frac{8}{13}$.

9. In like manner find the lacking term in each of the following, giving the explanation: —

1. $2 : 7 :: 5 : —$
2. $— : 4 :: 11 : 6$
3. $5 : 12 :: — : 8$
4. $34 : — :: 17 : 16$
5. $131 : 47 :: 1.5 : —$
6. $12\frac{1}{2} : 6\frac{2}{3} :: 8 : —$
7. $23.05 : 4.5 :: 7.1 : —$
8. $1.05 : 342 :: 100 : —$
9. $42 : 6 :: — : 30$
10. $\frac{2}{3} : \frac{4}{5} :: \frac{5}{8} : —$
11. $112 : 16 :: 49 : —$
12. $11\frac{3}{5} : 2\frac{1}{7} :: 4 : —$

RULE OF THREE.

410. The **Rule of Three** is an old term applied to the method of solving problems in which *Three Terms* of a proportion are given and the fourth is to be found.

1. If 8 yards of a certain kind of cloth cost \$35, how much will 42 yards of the same cloth cost?

It is evident that the same ratio exists between the cost of the two quantities as between the quantities, since the price per yard is the same. Hence the ratio of 8 yd. to 42 yd. is the same as the ratio of the cost of 8 yd., \$35, is to the cost of 42 yd. Stated as a proportion, this is 8 yd. : 42 yd. :: \$35 : the cost of 42 yd.

We have therefore to find the 4th term of the proportion $8 : 42 :: 35 : —$. This is $\frac{35 \times 42}{8} = \$183.75$. Hence 42 yd. will cost \$183.75.

2. If it require 12 bbl. of flour per year for a family of 10, how many barrels will it require for a family of 6?

The proportion is $10 : 6 :: 12 : —$. Hence we have $\frac{6 \times 12}{10} = 7\frac{1}{5}$.

Let the pupil give the reasons: 1st, for the statement of the proportion; 2d, for the method of finding the 4th term.

From the above analyses we deduce the following: —

411. Rule. — I. *Make that number the third term which is of the same kind as the answer sought.*

II. *If the nature of the problem requires the answer to be greater than the third term, make the* greater *of the two remaining terms the* second term; *otherwise make the* less *of those terms the second term: the term still remaining will be the* first *term.*

III. *Divide the product of the means by the given extreme, first indicating the operations in the form of a fraction, and then cancelling as much as practicable.*

NOTE. — In such questions there are always 3 quantities given, and *two of these are of the same kind.* Now it is necessary that we determine, from the nature of the case, whether the same ratio (relation) exists between the other two, one of which is *not* given, as exists between the two which are given. Not every problem in which three terms are given and a 4th required can be solved by proportion.

3. If 14 cords of wood cost \$98, what will 32 cords cost?

4. If 12 acres yield 384 bu. of wheat, what will 36 acres yield at the same rate?

What are the two quantities of the same kind? Does the same ratio exist between the quantities of wheat produced as between the quantities of land?

5. If the interest on \$350 at a certain rate and for a certain time is \$65, what is the interest on \$700 for the same time and rate?

Will *twice* as great a principal give *twice* as much interest at the same rate and for the same time? What are the two like terms? Are they not all alike? They are all *dollars.*

6. If \$100 principal amounts to \$122.50 for a certain rate and time, what principal does it require to amount to \$422.62½ for the same rate and time?

Amount. *Amount.* *Principal.* *Principal.*

$$\$122.50 : \$422.62\tfrac{1}{2} :: \$100 : \frac{42262.5}{122.5} \ (= \$345).$$

NOTE. — This is the question in true discount.

7. What is the present worth of a note \$422.62½, due 2 yr. 3 mo. hence, without interest, money being worth 10%?

Same as Ex. 6. \$100 *now* amounts to \$122.50 at the given rate and time. Hence the question is, How much *now* will it take to amount to \$422.62½ in the given time at the given rate?

8. What is the present worth of a note which amounts to \$350, 1 yr. 6 mo. 15 da. hence, money being worth 7%?

Find the amount of *any sum* for the given time and rate (\$100 is convenient). Then state and work the proportion.

9. What is the present worth of \$725.50, due in 6 yr. at 8%?

10. What is the present worth of \$2000, due in 3 yr. 6 mo., interest at 7%?

The examples in (383) can be solved by proportion, and may be reviewed and solved in this way.

11. A bankrupt paid 43 cents on every dollar of his debts. How much did he pay on a debt of \$569.31?

12. A difference of 15° in longitude makes an hour's difference in time. What is the difference in time between Boston, which is 71° 4′ 20″ W. long., and Washington, which is 77° 1′ 30″ W. long.?

13. If we measure a distance and find it to be 500 yd., measuring by a yard-measure which is afterward found to be one-eighth of an inch too short, what is the distance?

14. If a person by travelling 10 hours a day perform a journey in 31 days, in how many days will he perform the same journey if he travel 13 hours a day?

15. If a person perform a certain journey in 13½ days by travelling 10 hours a day, in what time will he perform the journey if he travel 11¼ hours per day?

16. Moll and Van Beek, in 1823, found that sound travels 332.05 meters in a second. What is the velocity per second in feet?

17. How far off is a battery when the flash precedes the report 15 sec., no allowance being made for the progressive motion of light?

18. It is found that the eclipses of Jupiter's moons occur 16 min. 26.6 sec. sooner when the earth is on the side of her orbit nearest Jupiter than when she is on the opposite side. The diameter of the earth's orbit being 183,000,000 miles, what is the velocity of light per second?

19. How many times would light go round the earth in a second, the earth's circumference being called 24,000 miles?

20. The nearest of the fixed stars are probably 100,000,000,000,000 miles from the earth. How long would one of them continue to be seen on the earth after it was annihilated, were such a thing possible?

21. If a staff 5 ft. long casts a shadow 3 ft., how high is a steeple whose shadow at the same time is 90 ft.?

PARTITIVE PROPORTION.

412. Partitive Proportion is a term applied to the division of a number into parts which shall be in the ratio of given numbers.

Ex. 1. — Divide 150 into three parts, which shall be to each other as 2, 3, and 5.

150 is the *sum* of three parts, which are to bear the ratio to each other of 2, 3, and 5, of which the *sum* is 10. Hence the proportions are, —

10 : 150 :: 2 : *the first part.*
10 : 150 :: 3 : *the second part.*
10 : 150 :: 5 : *the third part.*

2. Divide 35 into 2 parts, which shall be to each other as 3 to 4. As 2 to 5. As 1 to 6.

3. Divide 1 into 3 parts, which shall be to each other as $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{1}{5}$.

4. Divide 7 into 4 parts, which shall be to each other as 3, 5, 8, and 2.

5. Divide \$5000 among 3 persons so that the first shall have twice as much as the second, and the second twice as much as the third.

6. Divide \$100 into 4 parts, which shall be to each other as \$800, \$700, \$1000, and \$500.

7. Of \$1900 A is to have a certain sum, B twice as much, C twice as much as B, D as much as A and C, and E as much as B and D. How much is each to have?

8. A and B found a purse containing \$85, and agreed to share it in the ratio of $\frac{2}{3}$ to $\frac{3}{4}$. What did each receive?

9. A, B, and C entered into partnership. A put in \$340, B \$460, and C \$500. They gained \$390. What was the gain of each, the gain being divided in the ratio of the shares in the capital?

10. A, B, and C purchased a farm for \$3500, of which A furnished \$1500, B \$1500, and C \$500. They received \$280 rent for the farm. How much of this rent is due each?

11. A, B, C, and D hired a pasture for \$120. A put in 120 sheep, B 160, C 180, and D 140. How much ought each to pay?

---

12. Divide \$70 between A, B, and C, in such a manner that A's share shall be to B's as 2 to 3, and B's to C's as 4 to 5.

As often as A has \$2 B is to have \$3, and C is to have $\frac{5}{4}$ as much as B, or $\frac{\$15}{4}$. Hence we are to divide \$70 into three parts, which shall be to each other as 2, 3, and $\frac{15}{4}$, or as 8, 12, and 15.

13. Three persons in a joint speculation gain \$1000, which is to be divided so that the first share shall be to the second as 3 to 2, and the second to the third as 5 to 6. Required the shares.

COMPOUND PROPORTION.

413. **A Compound Ratio** is the product of two or more simple ratios, as a *Compound Fraction* is the product of two or more simple fractions.

Thus the ratio of 3 to 5 is $\frac{3}{5}$, and the ratio of 2 to 7 is $\frac{2}{7}$. Hence the compound ratio of 3 to 5 and 2 to 7 is $\frac{3}{5} \times \frac{2}{7}$, or $\frac{6}{35}$.

414. **A Compound Proportion** is an equality between a Compound Ratio and a Simple Ratio, the terms of each of the simple ratios being expressed.

If 240 is to some number which we wish to find, in the compound ratio of 3 : 4, 5 : 7, and 1 : 11, we write

$$\left.\begin{matrix} 3 : 4 \\ 5 : 7 \\ 1 : 11 \end{matrix}\right\} :: 240 : \text{the number sought.}$$

Ex. 1. — If 3 men, working 10 hours per day, can cut 51 cords of wood in 6 days, how much can 5 men cut in 7 days, working 8 hours per day?

The question is about the number of cords of wood.

First consider the number of men. 5 men will cut $\frac{5}{3}$ as much as 3 men, if they work 6 da. of 10 hr. each, or $\frac{5}{3}$ of 51 cd.

Second, in 7 da. the 5 men will cut $\frac{7}{6}$ as much as in 6 da., if they work 10 hr. per day, or $\frac{7}{6}$ of $\frac{5}{3}$ of 51 cd.

Third, but working 8 hr. per day, the 5 men will cut only $\frac{8}{10}$ as much as though they worked 10 hr.; i.e., —

$$\frac{8}{10} \text{ of } \frac{7}{6} \text{ of } \frac{5}{3} \text{ of } 51 \text{ cd.} = \frac{\overset{2}{\cancel{8}}_{4} \times 7 \times \cancel{5} \times \overset{17}{\cancel{51}}}{\underset{2}{\cancel{10}} \times \cancel{6} \times \underset{3}{\cancel{3}}} = \frac{119}{3} = 39\tfrac{2}{3} \text{ cd.}$$

In the form of a proportion this may be stated thus: —

$$\left.\begin{matrix} 5 : 3 \\ 7 : 6 \\ 8 : 10 \end{matrix}\right\} :: 51 \text{ cd.} : \text{the number of cords sought.}$$

From this analysis we deduce the following: —

415. **Rule.** — I. *Make that number the third term which is of the same kind as the answer sought.*

II. *Arrange the couplets of the compound ratio as in simple proportion, considering* one condition at a time.

III. *Divide the product of the means by the product of the given extremes, first indicating the operations in the form of a fraction, and then cancelling as much as practicable.*

2. If a footman can travel 150 miles in 5 days, when the days are 12 hours long, in how many days may he travel 275 miles when the days are 10 hours long?

3. If 5 oxen require an acre of grass for 9 days, how many acres will 20 oxen require for $30\frac{1}{2}$ days?

4. If 4 men eat 64 pounds of bread in 2 weeks, how many pounds will 16 men eat in 7 weeks?

5. If a man travel 100 miles in 3 days of 13 hours length, how far might he travel in 33 days of $14\frac{1}{4}$ hours length?

6. If 2 yards of cloth $1\frac{1}{2}$ yd. wide cost \$10.25, what cost 13 yards of like quality, which is $1\frac{3}{4}$ yd. wide?

7. If a family of 10 persons, in 2 weeks, spend \$200, how long ought a family of 13 persons to be in expending \$500?

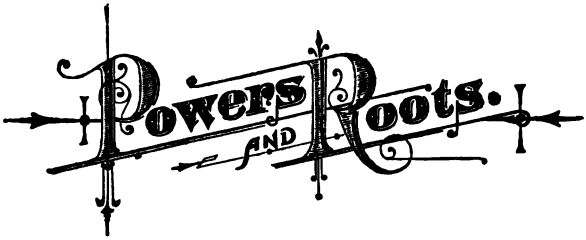
8. If 6000 lb. of bread will supply a garrison of 100 men 2 mo., how long will 12000 lb. last three such garrisons?

9. If 7 men can mow $84\frac{4}{5}$ acres in $12\frac{1}{2}$ days, working 8 hours per day, how many days of 10 hours each will 20 men require to mow $254\frac{2}{5}$ acres?

10. If 40 men in 8 days of 9 hours each build a wall 120 rd. long, 9 ft. high, and 3 ft. thick, how many men will be required to build a wall 162 rd. in length, 12 ft. high, and 9 ft. thick, in 18 days, by working 12 hr. each day?

11. If 100 men, by working 6 hr. each day, can in 27 da. dig 18 cellars, each 40 feet long, 36 ft. wide, and 12 ft. deep, how many cellars, each 24 ft. long, 27 ft. wide, and 18 ft. deep, can 240 men dig in 81 da. of 8 hr. each?

12. If 24 men, by working 8 hr. a day, can in 18 da. dig a ditch 95 rd. long, 12 ft. wide, and 9 ft. deep, how many men, in 24 da. of 12 hr. each, will be required to dig a ditch 380 rd. long, 9 ft. wide, and 6 ft. deep?



CHAPTER VII.

INVOLUTION AND EVOLUTION.

INVOLUTION.

416. A **Power** is the product arising from multiplying a number by itself a certain number of times.

Thus $3 \times 3 \times 3 = 27$; and 27 is called the 3d power of 3. $5 \times 5 = 25$; and 25 is the 2d power of 5. So $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$; and $\frac{8}{27}$ is the 3d power of $\frac{2}{3}$. The 2d power of .4 is .16. Of 1.2 is 1.44.

417. **Involution** is the process of raising a number to any required power. The number to be involved is called the *First Power*, or the *Root*.

418. The **Square** of a number is its 2d *power*, and the **Cube** of a number is its 3d *power*.

1. What is the square of 4? Of 7? Of $\frac{1}{2}$? Of $2\frac{1}{3}$? Of .35? The cube of 2? Of 10? Of 6? Of $1\frac{2}{3}$?

2. Write the squares and cubes of the 9 digits, and commit them to memory thoroughly.

419. A *Figure*, written at the right and a little above a number, indicates the power of that number. It is one form of what is called an *Exponent*.

Thus $4^2 = 4 \times 4$, or 16. $2^3 = 2 \times 2 \times 2$, or 8. $3^4 = 3 \times 3 \times 3 \times 3$, or 81. $.3^5 = .00243$. $1.1^3 = 1.331$.

3. What is the square of 23? Of 341? Of 3580? Of $2\frac{3}{4}$? Of 4.3? Of $\frac{6}{7}$? Of .35?

4. What is the cube of 6? Of 10? Of 1.5 Of $\frac{1}{2}$? Of .2? Of .04? Of $\frac{2}{5}$? Of $2\frac{2}{3}$? Of 1.07?

5. What is the value of these expressions: 2.1^2? 12^3? 7^4? $(\frac{2}{3})^2$? $.051^2$? $.01^3$? 1.01^2? 246^2? 10^5? 100^4? $(342)^2$? $(1834)^3$?

EVOLUTION.

420. A **Root** is one of the equal factors into which a number is conceived to be resolved. The *Square Root* of a number is one of *two* equal factors into which the number is conceived to be resolved. The *Cube Root* is one of three equal factors.

421. The *Radical* or *Root Sign* is $\surd$. When written thus, $\surd\overline{25}$, it indicates that the square root of 25 is to be taken; that is, that 25 is to be resolved into 2 equal factors, and one of them taken. To indicate the cube root, 3 is written in the sign. Thus $\sqrt[3]{\overline{125}}$ means the cube root of 125. It is 5.

$\surd 9$ is 3, because 3 is one of the 2 equal factors which compose 9.

$\sqrt[3]{343}$ is 7, because $7 \times 7 \times 7 = 343$.

1. What is the square root of 16? Of 36? Of 144? Of 81? Of 49? Of 1? Of 4? Of 9? Of 25? Of 121? Of 100? Tell why in each case.

2. What is $\sqrt[3]{8}$? $\sqrt[3]{27}$? $\sqrt[3]{1}$? $\sqrt[3]{1728}$? $\sqrt[3]{64}$? $\sqrt[3]{125}$? $\sqrt[3]{343}$? $\sqrt[3]{729}$? $\sqrt[3]{1000}$? $\sqrt[3]{216}$?

422. Finding the root of a number is called ***Extracting the Root.***

423. **Evolution** is the process of extracting roots.

424. A number is said to be a *Perfect Power* when it can be produced by multiplying some number by itself.

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425. ***Extraction of Roots of Perfect Powers.***

As Evolution is the process of finding one of a certain number of equal factors which compose a number, it is but a process of factoring, — resolving a number into equal factors.

Ex. 1. — Show what $\sqrt{16} =$. $\sqrt[5]{32} =$. $\sqrt[3]{1728} =$.

2. What is the square root of 1764?

Resolving 1764 into its prime factors, we find them to be 2, 2, 3, 3, 7, 7. Hence

$$2 \cdot 3 \cdot 7 \times 2 \cdot 3 \cdot 7, \text{ or } 42 \times 42 = 1764,$$

and 42 is the square root of 1764, being one of the two equal factors which compose it.

$$\begin{array}{r|l} 2 & 1764 \\ 2 & 882 \\ 3 & 441 \\ 3 & 147 \\ 7 & 49 \\ & 7 \end{array}$$

3. As above, find the square root of each of the following: 11025, 32400, 245025, 145600, 48841.

4. What is the cube root of 74088?

As above, resolving 74088 into its prime factors, we find $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 7$; i.e., $2 \cdot 3 \cdot 7 \times 2 \cdot 3 \cdot 7 \times 2 \cdot 3 \cdot 7$, or $42 \times 42 \times 42$. Hence $\sqrt[3]{74088} = 42$.

5. As above, find the cube root of each of the following: 46656, 621875, 18399744, 4741632.

General Method of Extracting the Square Root.

426. This method is based on the two following Principles: —

427. PRINCIPLE I. — *In squaring a number, the square of any order above units falls twice as many places to the left of units as the order itself.*

Thus the square of tens falls two places to the left, as $(50)^2 = 2500$; the square of hundreds falls four places to the left, as $(300)^2 = 90000$, or $(900)^2 = 810000$, etc.

428. PRINCIPLE II. — *The square of any number made up of tens and units is the square of the tens, + twice the product of the tens by the units, + the square of the units.*

Let us show this by squaring 68. Multiplying in the ordinary way, only writing each product by itself, we see at once that the square of 68 is $(6 \text{ tens})^2 + 2\,(6 \text{ tens} \times 8) + 8^2 = 4624$.

$$\begin{array}{r} 68 \\ 68 \\ \hline 8^2 = \quad 64 \\ 6 \text{ tens} \times 8 = \quad 48 \\ 8 \times 6 \text{ tens} = \quad 48 \\ (6 \text{ tens})^2 = 36 \\ \hline 4624 \end{array}$$

But it is necessary to prove this truth in a more general way, as it is the foundation of the very important *Rule for the Square Root.*

For this purpose, instead of using 8 for the units and 6 for the tens of the number we wish to square,

let u stand for *any number* of units,
and t for *any number* of tens.

Then, as 68 is 6 tens + 3 units, our number will now be represented thus, $t + u$.

We will now square $t + u$, observing, first, that tu means the same as $t \times u$, and that $2tu$ means twice tu. We multiply first by u, saying "u times u is u^2, and u times t is tu." Hence $t + u$ multiplied by u is $tu + u^2$. So t times $t + u$ is $t^2 + tu$. Adding these partial products (as in common multiplication), we have $t^2 + 2tu + u^2$. Which corresponds with the principle.

$$\begin{array}{r} t + u \\ t + u \\ \hline tu + u \\ t^2 + tu \\ \hline t^2 + 2tu + u^2 \end{array}$$

Ex. — Find the square of 56 according to Principle II. Of 87. Of 243. Of 3469. Of 43. Of 426. Of 1325.

$87^2 = (8 \text{ tens})^2 + 2\,(8 \text{ tens} \times 7) + (7^2) = 6400 + 1120 + 49 = 7569$. We may regard 243 as 24 tens and 3 units, and thus have $243^2 = (24 \text{ tens})^2 + 2\,(24 \text{ tens} \times 3) + 3^2$. Pupil, complete the work.

3469 may be regarded as 346 tens and 9 units, etc.

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429. Rule. — I. *Separate the number into periods by placing a point over units and over each alternate figure therefrom.*

II. *Write as the highest order in the root the square root of the greatest square in the left-hand period, subtract its*

square from that period, and to the remainder annex the next period, thus forming a new dividend.

III. *Double the root already found, regarding it as tens, as a Trial Divisor. By this divide the new dividend, annex the quotient to the root, and add it to the trial divisor, thus forming the True Divisor. Multiply the true divisor by the last root figure, subtract the product from the dividend, and to the remainder annex the next period.*

IV. *Repeat the process described in the last paragraph till the work is complete.*

When any trial divisor is not contained in the dividend, place a zero in the root, and also at the right of the divisor, and bring down the next period.

If any figure obtained for the root proves too large, diminish it by 1, and repeat the work.

Approximate roots may be obtained by annexing decimal periods of two zeros each. Decimal periods must always be full, since the square of any decimal has an even number of figures. Why?

Ex. 1. — Extract the square root of 4624.

OPERATION.

$(t+u)^2 = t^2 + 2tu + u^2 = \dot{4}6\dot{2}\dot{4}\ (68 = t+u.$

$t^2 = 36$

$2t = 120 \quad | \quad \overline{1024} = 2tu + u^2 = (2t+u)\,u.$

$u = \underline{\ 8} \quad |$

$2t+u = \overline{128} \quad | \quad \underline{1024} = (2t+u)\,u.$

EXPLANATION. — By placing a point over units figure, and over each alternate figure to the left, we see that the highest order in the root is *Tens*, according to Principle I., and that the square of the tens is in 46. Now the greatest square in 46 is 36. Hence 6 is the *tens* of the root.

Now letting $t+u$ represent the root, we have

$(t+u)^2 = t^2 + 2tu + u^2 = 4624$, by Principle II.

But having found t to be 6 (tens), we subtract its square from 4624, and have $2tu + u^2 = (2t+u)\,u = 1024$ remaining.

Now, as u is small with reference to $2t$, we may, *for a trial*, put $(2t) \times u$, or (12 tens) $\times\ u = 1024$. Hence 1024 ÷ 120 will give

the units figure of the root approximately at least. (In this case it gives it exactly.) Now, completing the divisor $2t + u$ by adding the units figure, we find the *True Divisor* to be 128. This multiplied by 8 gives 1024. Hence 68 is the exact square root of 4624.

QUERIES. — Why do we point off as we do the number whose square root is to be extracted?

Ans. — In order to ascertain what the highest order in the root will be, and what its square is, according to Principle I.

Why do we double the root already found, regarding it as tens, for a trial divisor?

Ans. — Because the remainder, which we use as a dividend, is approximately equal to twice this figure regarded as tens, multiplied by the next figure of the root.

Why do we add the root figure to the divisor?

Ans. — Because the *True Divisor* is twice the root previously found regarded as tens + the figure last found.

Why do we bring down only one period at a time?

Ans. — Because the square of the root as far as found in that step can form no part of the remaining periods. Thus, when we are finding the hundreds figure of the root, we do not need the two right-hand periods, since the square of hundreds falls beyond four places from units (Prin. I.).

2 and 3. Extract the square root of 74529, and 2125764.

OPERATION. — When there are more than two figures in the root.

$\dot{7}45\dot{2}9$ (273	$\dot{2}1\dot{2}5\dot{7}6\dot{4}$ (1458
4	1
47) 345	24) 112
329	96
543) 1629	285) 1657
1629	1425
	2908) 23264
	23264

NOTE. — The same explanation which has been given when the root consists of two figures can be readily extended to any number. Thus, in extracting the square root of 74529, the square of the first two figures 27 (tens) falls in orders from hundreds upwards. We may therefore proceed to find these two figures exactly as if we were extracting the root of 745. Having found these, we may take them as tens, and consider the root as made up of 27 (tens) and some *number* of units, etc.

4–12. Extract the square root of 2209, 361, 2601, 4900, 120409, 412164, 123201, 6718464, 966289.

13–15. Extract the square root of 87.512. Of 2. Of .4.

OPERATIONS.

$\dot{8}\dot{7}.5\dot{1}2\dot{0}$ (9.354+	2 (1.4142+	.40 (.632+
81	1	36
183) 651	24) 100	123) 400
549	96	369
1865) 10220	281) 400	1262) 3100
9325	281	2524
18704) 89500	2824) 11900	576
74816	11296	
4684	28282) 60400	

All that is peculiar in extracting the roots of numbers wholly or in part decimals is to make the number of decimals even by annexing a 0 where there is an odd number of significant figures. This arises from the fact that the square of any decimal must have an even number of figures, since there is as many decimals in the quotient as in both factors. We point off, as directed in the rule, by placing a point over each alternate figure from units.

16–25. Extract the square root of .0256, 5.32, 28.6, 34.3, 7.5, 4.9, .0049, .049, 582.431, 3.671, extending the root to three places of decimals when it is not exact.

26–36. Extract the square root of $\frac{49}{121}$, $\frac{144}{515}$, $\frac{2}{9}$, $\frac{3}{4}$, $\frac{5}{11}$, $\frac{128}{347}$, $\frac{43}{87}$, $1\frac{2}{3}$, $2\frac{1}{4}$, $1\frac{11}{25}$, $3\frac{2}{5}$.

Since a fraction is involved by raising numerator and denominator separately to the required power, its root may be extracted by extracting the root of each term separately.

If neither of the terms is a perfect power, it is best to reduce the fraction to a decimal, extending the operation to such degree of accuracy as may be desired.

When the denominator is a perfect power, and the numerator is not, extract the root of the denominator, and also of the numerator, extending the work as far as desired, and then divide the latter by the former.

General Method of Extracting the Cube Root.

430. The general rule for extracting the cube root is based on the two following principles: —

431. PRINCIPLE I. — *In cubing any number, the cube of any order above units falls three times as many places to the left as the order itself.*

Thus the cube of 3 tens, or $(30)^3 = 27{,}000$. The cube of 5 hundreds $(500)^3 = 125{,}000{,}000$.

432. PRINCIPLE II. — *The cube of any number made up of tens and units is the cube of the tens, + 3 times the square of the tens multiplied by the units, + 3 times the tens multiplied by the square of the units, + the cube of the units.*

DEMONSTRATION. — Let the number we propose to cube be represented by $t + u$, as in (**492**). Now the square of $t + u$ is by (**492**) . . .

$$t^2 + 2tu + u^2$$

Multiplying this by $t + u$, we have the cube.

$$\begin{array}{r} t^2 + 2tu + u^2 \\ \hline t + u \\ \hline t^2u + 2tu^2 + u^3 \\ t^3 + 2t^2u + tu^2 \\ \hline t^3 + 3t^2u + 3tu^2 + u^3 \end{array}$$

The multiplication is explained thus: multiplying by u, we have u^2 multiplied by u, which makes u^3, just as 2 squared (2^2) multiplied by 2 makes 2 cubed (2^3). $2tu$ multiplied by u makes $2tu^2$; for $2tu$ is $2 \times t \times u$, and, putting in another factor of u, we have $2 \times t \times u \times u$, or $2tu^2$. In like manner the other terms are multiplied. In adding the partial products we notice that 2 times tu^2 and 1 time tu^2 make 3 times tu^2, or $3tu^2$. So 2 times t^2u and 1 time t^2u make $3t^2u$. Finally, we observe that this result, $t^3 + 3t^2u + 3tu^2 + u^3$, agrees with our statement of the principle.

433. Rule. — I. *Separate the number into periods by placing a point over units, and over each third figure therefrom.*

II. *Write as the highest order in the root the cube root of the greatest cube in the left-hand period, subtract its cube from that period, and to the remainder annex the next period, thus forming a new dividend.*

III. *Take 3 times the square of the root already found, regarded as tens, for a Trial Divisor. By this divide the new dividend, which will give approximately the next figure in the root.*[1] *Then form the True Divisor by adding to the Trial Divisor 3 times the product of the preceding part of the root (regarded as tens) by the last figure, and the square of this last figure. Multiply this True Divisor by the last root figure, subtract the product from the last dividend, and to the remainder annex the next period.*

IV. *Proceed as described in the last paragraph till the work is complete.*

[See notes under rule for Square Root.]

1. Extract the cube root of 262144.

OPERATION.

$(t+u)^3 = t^3 + 3t^2u + 3tu^2 + u^3 = 262\dot{1}44\dot{}$ $(64 = t + u.$

$t^3 = 216$

$3t^2 =$	10800	$46144 = 3t^2u + 3tu^2 + u^3$, or
$3tu =$	720	$(3t^2 + 3tu + u^2)\ u.$[2]
$u^2 =$	16	
$3t^2 + 3tu + u^2 =$	11536	$46144 = (3t^2 + 3tu + u^2)\ u$, or
		$3t^2u + 3tu^2 + u^3.$

EXPLANATION. — Pointing off into periods, by placing a point over units and over each third figure therefrom, we see that the highest order in the root is tens (Principle I.), and that its cube is contained in 262. Now the greatest cube in 262 is 216, the cube root of which is 6. Hence 6 is the tens figure of the root.

Now, letting $t+u$ represent the root, we have $(t+u)^3 = t^3 + 3t^2u + 3tu^2 + u^3 = 262144$.

But, having found t to be 6 (tens), we subtract its cube from 262144, and have $3t^2u + 3tu^2 + u^3 = (3t^2 + 3tu + u^2)\ u = 46144$ remaining.

Now, as u is small with reference to $3t^2$, we may, *for a trial*, put

[1] The root figure thus found may be too large; but, if so, the fact will appear when the True Divisor is multiplied by it.

[2] This means $3t^2 + 3tu + u^2$ multiplied by u, which makes $3t^2u + 3tu^2 + u^3$.

$(3t^2) \times u = 46144$, and, as we know t, this becomes $10800 \times u = 46144$. Hence u is, approximately, $46144 \div 10800$.

We thus find that the units figure is probably 4. Now, completing the divisor by adding to it $3tu$, — i.e., 3 times the product of the root already found (remembering that it is tens) by the units, — and the square of the units, we find that the *True Divisor*, $3t^2 + 3tu + u^2$, is 11536. This multiplied by 4 gives 46144. Hence 64 is the exact cube root of 262144.

[Similar queries to those following the explanation of the method of extracting the square root may be raised here, and can be answered in a similar manner from the above explanation.]

2–5. Extract the cube root of 54872. Of 41063625. Of 354894912. Of 3416.53.

54872 (38	41063625 (345
27	27
2700 | 27872	2700 | 14063
720	360
64	16
3484 | 27872	3076 | 12304
	346800 | 1759625
	5100
	25
	351925 | 1759625
354894912 (708	3416.530 (15.06
343	1
[1]14700 | 11894912	300 | 2416
1470000	150
16800	25
64	475 | 2375
1486864 | 11894912	6750000 | 41530000
	27000
	216
	6777216 | 40663296
	866704

6–13. Extract the cube root of the following: 157464; 74088; 571787; 15625; 2744; 1124864; 2571353; 651714363.

[1] As this is not contained in 11894, we write 0 in the root, and bring down the next period.

14–26. Extract the cube root of 34285.7; 34.3472; 5; 48; 2; .4932; .8; .343; .27; $\frac{125}{256}$; $\frac{343}{729}$; $\frac{28}{125}$; $\frac{14}{33}$.

[See note under Ex. 26–36 in Square Root.]

Applications.

434. PROBLEM 1. — *Given the area of a square, to find its side.*

RULE. — Extract the square root of the area.

The reason for the rule is, that the area of a square is the square of its side, as a square is a rectangle whose sides are equal to each other. (**215.**)

NOTE. — If the area is given in a denomination which has no corresponding linear unit (as in *acres*), it should be reduced to a denomination which has such a linear unit (as to *rods*), before the root is extracted.

1. What is the side of a square which contains 74529 square feet?

2. How many rods on a side is a square acre? How many feet?

3. How many rods on a side is a square 40-acre lot? A square 80-A.?

4. What is the side of a square containing 10 Ars? What of one containing 40^{a}? 60^{a}? 100^{a}? 25^{Ha}? 70^{Ha}?

5. A man has a farm containing 520 acres, in the form of a rectangle, twice as long as wide. What are the dimensions?

6. What is the side of a square which contains twice as much as one 1 ft. on a side? 3 times as much? 4 times? 10 times? 9 times?

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435. When *three* numbers are so related that the 1st is to the 2d as the 2d is to the 3d, the 2d is called a **Mean Proportional** between the other two. The third term is called a *Third Proportional to the other two.*

Thus, in the proportion 2 : 4 :: 4 : 8, 4 is a *Mean Proportional* between 2 and 8, and 8 is a *Third Proportional* to 2 and 4.

436. PRINCIPLE. — *A Mean Proportional between two quantities is the square root of their product.*

Thus let a stand for the *first* number, b for the *third*, and m for the *mean;* then $a : m :: m : b$, and $m^2 = ab$, or $m = \sqrt{ab}$.

EX. 1. — Find the mean proportional between 4 and 9. 27 and 3. 5 and 11. $\frac{2}{3}$ and $\frac{1}{2}$.

2. Find a third proportional to 7 and 4. To 8 and 12. To 12 and 8. To 4 and 6. To 6 and 4. To $\frac{2}{3}$ and $\frac{1}{5}$. To $\frac{3}{5}$ and $\frac{2}{3}$.

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437. PROBLEM 2. — *Given the contents, to find the edge of a cube.*

RULE, — Extract the cube root of the contents.

[Reason and note similar to the above.]

1. What is the edge of a cube containing 45,499,293 cu. ft.? One containing 487.42 cu. ft.?

2. What must be the edge of a cubical box which shall contain a bushel? (See **229.**) What of one containing 100 bushels?

3. How many feet on an edge is a cubical mass containing 389,017 cu. ft.?

4. How many feet on an edge is a cube whose content is a cord?

5. Mr. A. received $97.20 for digging a cellar at 20¢ per cubic yard. The cellar was 3 times as wide as it was deep, and twice as long as it was wide. What were its dimensions in feet?

6. The edge of a cubical vessel is 1 ft. What is the edge of one which contains 2 times as much? 3 times? 4 times? 8 times? 27 times? 40 times?

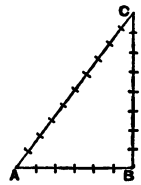
CHAPTER VIII.

PROBLEMS IN MENSURATION.

[For definitions of point, line, angle, rectangle, square, solid, parallelopiped, and cube, see *Measures of Extension*, p. 171, *et seq.*]

438. PROBLEM 1. — *To find one side of a right-angled triangle when the other two are given.*

439. A **Triangle** is a plane (flat) figure with only 3 sides. When one of its angles is a right angle, it is called a *Right-Angled Triangle.* If you make a right-angled triangle whose *Base*, **AB**, is 6, and whose *Perpendicular*, **CB**, is 8, and then measure the *Hypothenuse*, **AC**, you will find the latter to be just 10. Now, notice that $6^2 + 8^2 = 10^2$, and in Geometry this is found true of all right-angled triangles. *The square of the hypothenuse equals the sum of the squares of the other sides.*



440. Since $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$, $AC = \sqrt{\overline{AB}^2 + \overline{BC}^2}$;

Also, $\overline{AB}^2 = \overline{AC}^2 - \overline{BC}^2$, $AB = \sqrt{\overline{AC}^2 - \overline{BC}^2}$;

And $\overline{BC}^2 = \overline{AC}^2 - \overline{AB}^2$, $BC = \sqrt{\overline{AC}^2 - \overline{AB}^2}$.

[It is important that the pupil become familiar with this form of statement, and be able to put it into common language.]

Ex. 1. — What is the length of a brace in a building which goes into the post 3 ft. from the angle, and into the beam 2 ft. from the angle?

2. What is the length of a brace which reaches from corner to corner of a gate which is 10 ft. long and 4½ ft. high?

3. What length of rafter is required for a building 30 ft. wide, the ridge of the roof being 10 ft. higher than the plates, making no allowance for the projection beyond the plates?

4. What is the distance from one corner on the floor of a room to the diagonally opposite corner in the ceiling, the room being 18 ft. by 20 ft., and 12 ft. high?

5. Wishing to find the distance between two trees between which a pond lay, I measured from one of the trees a line perpendicular to the line joining the trees, a distance of 100 rods, and from the end of this line measured a right line to the other tree, 160 rods. What was the distance between the trees?

6. Having a pole which I knew to be 30 ft. long, and wishing to find the height of a wall, I put one end of the pole against the top of the wall, and found that the other end rested on the ground 12 ft. from the wall. What was the height of the wall?

7. What must be the length of the rafter for a *quarter-pitch* roof on a house 32 ft. wide? What for a third-pitch roof? Half-pitch? Whole-pitch?

Quarter-pitch means that the height of the ridge above the plates is ¼ the span, etc.

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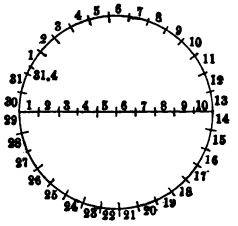
441. A **Circle** *is a plane (flat) figure bounded by a curved line, every point of which is equally distant from a point within called the* **Centre.** *The curved line is called the* **Circumference.** *A line drawn from the centre to the circumference is called a* **Radius.** *A line drawn through the centre and terminated by the circumference is a* **Diameter.**

442. PROBLEM 2. — *To find the circumference of a circle when the diameter is given.*

By drawing a circle very carefully, say 1 inch in diameter, as in the margin, and dividing the diameter into 10ths inches, a nice pair of dividers can be opened one 10th inch, and made to step around the circumference. If it is all done with nicety, it will be found to be a little over 31 steps around when it is 10 across. Hence we can learn that the circumference is a little more than 3.14 times the diameter. *In Geometry it is shown that the circumference is very nearly* 3.1416 *times the diameter.* Nobody knows the exact ratio: it is represented in Geometry by the Greek letter π (pi).



Ex. 1.—How long a bar of iron will it take to make a tire for a wheel which is 50 in. in diameter?

2. What is the diameter of a tree which measures $33\frac{1}{3}$ ft. around?

3. As the earth revolves on its axis once in 24 hours, at what rate per hour do we move in consequence of this rotation, the radius of the earth being 3962 miles?

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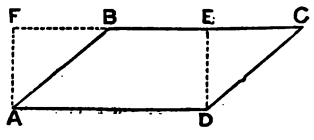
443. Parallel lines *are lines which run in the same direction. A plane figure bounded by four sides is a* **Quadrilateral.** *When each side is parallel to the side opposite, the figure is a* **Paralellogram**; *and, when the angles are all right angles, the figure is a* **Rectangle.**

444. PROBLEM 3.—*To find the area of a parallelogram.*

Let ABCD be any parallelogram. Draw DE perpendicular to AD, and consider the triangle EDC as cut off, and placed on the other end in the position AFB. Then the rectangle AFED has the same area as the parallelogram ABCD. But the area of the rectangle is the product of the base and altitude (215).



Hence the area of any parallelogram is the product of its base and altitude.

Ex. 1. — A piece of velvet is cut "on the bias" at both ends; i.e., diagonally across. If the piece is $\frac{1}{2}$ yd. wide, how much velvet is there in a piece which measures 8 yd. along the selvage, the entire figure being a parallelogram?

2. How many square yards in the plastering of a ceiling of a room 20 ft. by 18?

3. There are two parallel roads $\frac{1}{2}$ a mile apart. Mr. A.'s farm runs obliquely across from one to the other in the form of a parallelogram. How many acres in his farm if it measures 1 mi. along the road?

4. How many bricks (a common brick is $2 \times 4 \times 8$ in.) laid on the side will it take to pave a cellar 15×18 ft. on the bottom? How many if laid on the edge?

5. How many tiles 6 in. square will lay a floor 20×36 ft.?

6. What is the cost of plastering a room 20×24 ft. and $10\frac{1}{2}$ ft. high, including the ceiling, at 25¢ per square yard, there being 3 doors $7\frac{1}{2}$ ft. high and 3 ft. 9 in. wide, and 4 windows 6 ft. high and $3\frac{1}{2}$ ft. wide?

7. How many yards of cloth 27 in. wide will it take to line 12 yd. $1\frac{1}{4}$ yd. wide?

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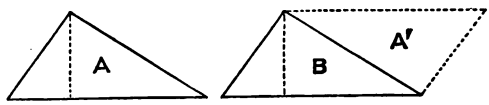
445. PROBLEM 4. — *To find the area of a triangle.*

Suppose A is the triangle whose area we seek. Make another just like it, as A′, and then put the two together, as B and A′. Then the two make



a parallelogram, whose area is the product of the base and altitude. But the triangle is half the parallelogram. *Hence the area of a triangle is half the product of the base and altitude,* or half of either *multiplied* into the other.

446. NOTE. — If the three sides only are given, *from half the sum of the three sides subtract each side separately, multiply the half sum and these remainders together, and extract the square root of the product.*

The pupil cannot see *why* this is so without considerable knowledge of Geometry.

1. What is the area in acres of a triangle whose base is 40 rods and altitude 24 rods?

2. What is the area in acres of a triangular field whose base is 85 rods and altitude 75 rods?

3. How many square inches in a triangular board whose sides are respectively 12, 18, and 24 inches?

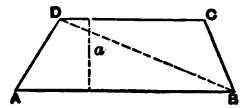
4. How many acres in a triangular field whose sides are respectively 20, 30, and 40 rods?

5. What is the area in acres of a triangular piece of ground whose base is 7.52 ch., and altitude 5.32 ch.?

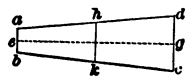
---

447. PROBLEM 5. — *To find the area of a trapezoid.*

A **Quadrilateral** is a plane figure with four sides. If two of these are parallel, and the other two not, the figure is a **Trapezoid.** ABCD is a trapezoid, and *a* is its altitude. It is evident that the area is the area of two triangles having the same altitude as the trapezoid, and, for their bases, the upper base and the lower base. *Hence the area of a trapezoid is ½ the product of the altitude into the sum of the bases,* or half the sum of the bases into the altitude.



Let *abcd* be a tapering board. Such a board is a trapezoid, and *ab* and *cd* are the bases, and *eg* the altitude. Now, the width across the middle, as *hk*, is half the sum of the two bases, or ends. *Hence, to find the area of such a board, multiply its width, measured in the middle, by its length.*



1. How many acres in a trapezoid whose parallel sides are 45 and 60 rods, and the distance between them 50 rods?

2. How many square feet in the surface of a board 14 ft. long, and 21 in. wide in the middle?

448. *In measuring boards the* SURFACE MEASURE *alone is considered, unless the board is more than an inch thick.*

When the lumber is 1¼ in. thick, ¼ is to be added to the superficial measure; when 1½ in. thick, ½ is to be added; when 2 in. thick, the superficial measure is to be doubled. Why?

QUERY. — Why is it that in measuring boards we may multiply the length in FEET by the width in INCHES, and divide the product by 12 to get the square feet?

3. How many square feet in a load of boards in which there are 10 boards 12 ft. long and 10 in. wide, 8 boards 16 ft. long and 8 in. wide, 20 boards 14 ft. long and 10 in. wide, the widths all being measured in the middle? How much would this load amount to at $14 per M?

4. How many feet of boards in a load consisting of 30 pieces of 1¼ in. stuff, 12 ft. long 6 in. wide; 40 pieces of 1½ in. stuff, 10 ft. long and 9 in. wide; and 20 bolts of "siding" (½ in. stuff), 12 ft. long and 5 in. wide, with 6 pieces in each bolt?

5. How much flooring 1¼ in. thick will it take for the floors of 10 rooms, 2 being 18 × 15 ft. (18 ft. by 15 ft.) each, 2 15 × 12 ft. each, 1 12 × 16 ft., 1 12 × 12 ft., 2 10 × 12 ft. each, and 2 10 × 10 ft.? How much will it cost at $18 per M?

6. My house-lot is a corner-lot 4 rods by 8. What will the lumber cost for a 2 in. plank-walk, 6 ft. wide, at $18 per M, with 3 by 4 in. scantling for stringers, the stringers to be laid crosswise, and so that each 12 ft. plank shall rest on 4 stringers, the scantling costing $12 per M?

7. Carpenters often charge by the *square* (10 by 10 ft.) *for laying* floors, ceiling, roofing, etc. What will the floor-

ing of four rooms with $1\frac{1}{4}$ in. plank cost at \$20 per M for the plank, 75¢ per square for laying, and 20 lb. of nails at 3¢ per lb., the rooms measuring respectively 18 ft. square, 15 by 18 ft., 12 by 15 ft., and 16 ft. square?

8. Mr. A. has a farm, two of whose sides are north and south lines $\frac{1}{2}$ a mile apart. One of these sides is 60 rods, and the other 80. How many acres in his farm?

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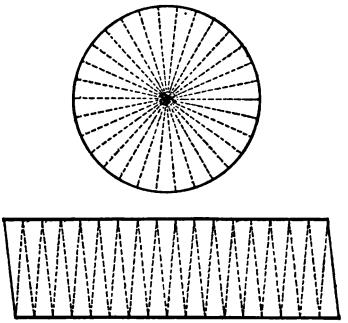
449. PROBLEM 6. — *To find the area of a circle.*

If you cut out of a tough piece of cardboard a circle, then cut it in two on a diameter, and cut from the centre through *almost* to the circumference, as represented by the dotted lines, you can straighten the semicircumferences out, and make two comb-like pieces, which you can slip together and thus make a very perfect parallelogram. From this you will see that *the area of a circle is the product of its radius into its semicircumference.*



Now, if the radius is r, we have learned that the circumference is $3.1416 \times 2r$, and the semicircumference $3.1416 \times r$. If we multiply this by r, we have $3.1416 \times r^2$. Thus we see that *the area of a circle is the square of the radius multiplied by* 3.1416.

1. What is the area in square feet of a circle whose diameter is 28 inches? What is the circumference?

2. Two 5 in. stove-pipes run into a 7 in. pipe. Is the 7 in. pipe large enough to carry the smoke of the two?

3. How many square feet in a circle 5 ft. 6 in. in diameter?

4. What is the area in acres of a circular piece of ground whose circumference is 1 mile?

5. A horse is tied by a rope to a stake in a meadow. The rope being attached to his head, how long must it be so that he can graze over an acre?

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450. PROBLEM 7. — *To find the amount of carpeting required for a room whose dimensions are given.*

In order to do this, we must know which way the breadths are to run, the width of a breadth, and how much will be wasted on a breadth in cutting so as to match the figures. Often it is necessary to know whether it is practicable to split a breadth. When these facts are known, *we find the number of breadths which will be required, and the length of a breadth including waste, and multiply the two together.*

1. How much carpeting, $\frac{3}{4}$ yd. wide, will it take to carpet a room 18 ft. by 15 ft. if the breadths run lengthwise, and it wastes 6 in. on a breadth in matching? How much, if the breadths run crosswise, and it wastes 2 in. in matching?

2. What will it cost to carpet a room 14 ft. by 17 ft. with yard-wide carpeting at 89¢ per yard, which wastes 4 in. on a breadth in matching, if the breadths run crosswise? What, if the breadths run lengthwise, and it wastes 6 in. in matching?

3. Wall-paper is usually 18 inches wide, and 8 yards make a roll. How many rolls must we buy (we can buy only *whole* rolls) to paper a room 16 × 18 ft. whose walls are 10 ft. high, no allowance being made for doors and windows? What will it cost at 40¢ per roll if an allowance of $\frac{1}{4}$ be made for doors and windows?

Find how many strips of 10 ft. each it takes.

4. How many rolls of wall-paper must we buy to paper 2 rooms, one 12 × 16 ft. and the other 15 × 18, each 10 ft. high, there being 7 doors and windows in one room, and 6 in the other, each of which is $3\frac{1}{2}$ ft. wide; it being understood *that the* strips are not to be pieced, but that the 4 ft. which

each roll will overrun will piece out above the doors and windows?

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451. Similar Figures are such as have the same shape.

452. *The areas of similar plane figures are in the same ratio as the squares of their like sides, or lines.*

1. There are two fields of the same shape (similar) : one is 10 rods on a certain side, and the other is 15 rods on the corresponding side. What is the ratio of their areas?

2. Two circles have their diameters respectively 6 and 8. What is the ratio of their areas?

453. *The volumes of similar solids are in the same ratio as the cubes of their corresponding edges, or lines.*

1. How many times as large as a 2 in. ball is a 3 in. ball?

2. A cubical bin 5 ft. on an edge contains (practically) 100 bu. How much does one 6 ft. on an edge contain? 10 ft.? 4 ft.?

3. Which is the stouter (bulkier, "fleshier"), a man 5 ft. 10 in. in height who weighs 175 lb., or one 6 ft. who weighs 180 lb.?

4. In the last example, what must be the weight of the 6 foot man in order that he may have the same proportions as the other?

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454. PROBLEM 8. — *To find the volume of a right parallelopiped.*

(See **216.**)

1. How many cubic feet in a pile of 4 foot wood (that is, sticks of wood 4 feet long) piled in the usual way, the pile being 32 feet long and 5 feet high? How many cords?

2. How many cords of wood in a pile of 4 foot wood 130 ft. long and 6 ft. high?

SUGGESTION. — A convenient way to solve such an example is this:—

$$\frac{\overset{65}{\cancel{130}} \times \cancel{4} \times \overset{3}{\cancel{6}}}{\cancel{4} \times \cancel{4} \times 8} = \frac{195}{8} = 24\tfrac{3}{8}.$$

3. How many cords in a pile of 6 ft. wood 148 feet long and 8 feet high?

4. How many cords in a pile of wood 36 ft. long, 4 ft. wide, and 4 ft. high? When wood is piled in this way, how many feet in length of the pile does it take to make a cord?

5. Why does $\frac{250}{8}$ give the number of cords in a pile of 4 ft. wood 4 ft. high and 250 ft. long?

6. How many cords in three piles of 4 ft. wood, the first 36 ft. long and 4 ft. high, the second 42 ft. long and 5 ft. high, and the third 20 ft. long and 6 ft. high?

7. How many cords in a pile of 4 ft. wood 200 ft. long and 7 ft. high? How many, if the wood be 3 ft. long? If 2 ft. long? How many in each case, if the pile be 6 ft. high? 5 ft. high? 8 ft. high?

8. How much wood in a load consisting of 3 lengths of 4 foot wood, the load being 3 ft. 2 in. wide and 2 ft. 6 in. high?

9. How much wood in a load consisting of 2 lengths of 4 ft. wood, the average width of the load being 2 ft. 9 in. and the height 3 ft.?

10. How many cubic yards in a ditch $\frac{1}{2}$ mi. long, and which averages 3 ft. deep and 4 ft. wide?

11. At 20¢ per cubic yard what does it cost to excavate a cellar 20 ft. by 30 ft., and $6\frac{1}{2}$ ft. deep?

12. At 30¢ per cubic foot what cost a stick of timber $15\frac{1}{2}$ in. square and 20 ft. long?

13. What is the cost of digging a trench 650 ft. long, $2\frac{1}{2}$ ft. wide at top, and $1\frac{1}{2}$ ft. wide at bottom, and averaging $3\frac{1}{4}$ ft. deep, at 25¢ per cu. yd.?

The average width is 2 ft.

14. The end (cross-section) of a certain railroad tunnel is 525 sq. ft., and the length of the tunnel $\frac{3}{8}$ of a mile. How many cubic yards were removed in the excavation?

15. In a room 125 ft. by 90 ft., with ceiling 30 ft. high, and seating 2500 persons, how soon will the air have been all breathed, allowing 10 cu. ft. per minute for each person?

16. Milwaukee bricks are $8\frac{1}{2} \times 4\frac{1}{2} \times 2\frac{3}{8}$ in. If it takes 40000 common bricks ($8 \times 4 \times 2$ in.) for a particular structure, how many Milwaukee bricks will it take?

17. Philadelphia and Baltimore bricks are $8\frac{1}{4} \times 4\frac{1}{8} \times 2\frac{3}{8}$ in. How many such bricks would be required for the structure in Ex. 16?

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455. Doyle's Rule for calculating the amount of square-edged inch boards which can be sawed from a round log is this: *From the diameter in inches subtract 4; the square of the remainder will be the number of square feet of inch boards yielded by a log 16 ft. in length.*[1]

The yield of logs of the same diameter is in the ratio of their lengths. The log is scaled; i.e., its diameter is measured, at the top end.

1. What is the board measure of a log 16 ft. long and 18 in. in diameter? One 20 in. in diameter? 30 in.?

2. What is the board measure of a log 12 ft. long and 2 ft. in diameter? 31 in.? 40 in.? 37 in.?

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[1] This rule, so admirable in its simplicity, is the foundation of the table in *Scribner's* popular *Lumber and Log Book*, which is said to have a larger sale than all other books of the kind together, and is a generally recognized standard among lumbermen. Nevertheless, in a scientific point of view, the rule is but a rude approximation. See the Author's SCIENCE OF ARITHMETIC, p. 277.

460. When the base of a Right Pyramid, or of a frustum, passes into a circle, the solid becomes a Right Cone, or frustum thereof.

461. PROBLEM 10.—*To find the area of the convex surface of a right pyramid or cone.*

Multiply the perimeter of the base by ½ the slant height.

462. PROBLEM 11. — *To find the area of a frustum of a right pyramid or cone.*

Multiply ½ the sum of the perimeters of the bases by the slant height.

463. PROBLEM 12. — *To find the volume of a right pyramid or cone.*

Take ⅓ the product of the base and altitude.

464. PROBLEM 13. — *To find the volume of a frustum of a right pyramid or cone.*

Multiply the sum of the two bases and a mean proportional between them by ⅓ the altitude.

1. What is the convex surface of a right pyramid whose base is a polygon of 5 sides, each of which is 10 ft., and whose slant height is 14 ft.?

2. What is the volume of a pyramid whose base is a square 200 feet on a side, and whose altitude is 150 feet?

3. How many barrels will a cistern in the form of an inverted frustum of a right cone contain, the diameter of the bottom being 8 ft., of the top 10 ft., and depth 7 ft.?

465. PROBLEM 14. — *To find the area of the surface and the volume of a sphere whose radius is given.*

The area of the surface is 4 times the area of a circle with the same radius.

The volume is the product of $\frac{1}{3}$ the radius into the area of the surface.

1. Find the area of the surface and the volume of the earth, its diameter being 7916 miles.

2. What is the area of the surface, and what the volume, of a sphere 2 ft. in diameter?

STRENGTH OF BEAMS.

466. *The* STRENGTH *(power to support weight) of rectangular* BEAMS, *supported at both ends, is in the ratio of their cross-sections (ends), multiplied by their depths.*

1. How much stronger is a 3 by 4 in. beam when set on edge than when lying flat? A 2 by 8? A $2\frac{1}{2}$ by 10?

2. Which is the stronger beam, one 6 in. square, or one $2\frac{1}{2}$ by 10 in., set on edge? What is the ratio of their strengths?

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467. THERMOMETER SCALES.

1. The scale of the common thermometer (*Fahrenheit*) is divided into 180° between the freezing point (32°) and the boiling point (212°). The *Centigrade* scale is divided into 100°, from freezing (0°) to boiling (100°). What is the relative length of the degrees?

2. When the temperature is 68° F., what is it Centigrade? When 85° F., what is it C.? When 20° below 0, or —20° F., what is it Centigrade?

3. 29° C. is what F.? 30° C.? —18° C.? —15° C.?

CHAPTER IX.

468. ONE HUNDRED TEST EXERCISES IN ARITHMETICAL OPERATIONS.

1. $3\frac{1}{2} - \frac{2}{4} + \frac{4}{\frac{1}{5}}$.

2. $\frac{3}{5}$ of $4\frac{2}{5} \times .25$.

3. $1.33\frac{1}{3} \times 4\frac{1}{5} \div \frac{2}{\frac{3}{5}}$.

4. $\frac{11\frac{2}{3}}{12\frac{2}{5}} + \frac{57\frac{1}{2}}{\frac{3}{16}} - \frac{2}{\frac{1}{4}}$.

5. $(5\frac{1}{2} + 3) \times (2 - \frac{3}{7})$.

6. $\frac{4 - \frac{2}{3}}{3} - \frac{3\frac{1}{2} + 2}{8}$.

7. $\frac{5\frac{1}{3} + 2\frac{3}{4}}{4.5 - 1\frac{1}{3}} \div 6.25$.

8. $\frac{1\frac{1}{4} \times \frac{4}{3}}{5\frac{1}{2}} \div \frac{3 - \frac{3}{5}}{\frac{1}{2} \text{ of } 7}$.

9. $\frac{4\frac{1}{3} + 2\frac{1}{5} - 3}{\frac{2}{5} \text{ of } 1.25} \times (\frac{2}{3} + \frac{1}{2})$.

10. $\frac{5.8 \div .002}{1.6} + \frac{.45}{5}$.

11. $\frac{1 \div .0001}{.5 \div 50} \times 400$.

12. $\frac{42.68 \div .002}{\frac{1}{5} \text{ of } 13} \div .8$.

13. $\frac{500 \div (\frac{2}{3} \text{ of } .066)}{(\frac{1}{2} \text{ of } .7) \div (\frac{4}{5} - \frac{2}{20})}$.

14. $\frac{.3 + .03 + .003}{3.5 \div .07}$.

15. $\frac{56 \div .007}{\frac{1}{2} \text{ of } .04} \div \frac{.02}{20}$.

16. $\frac{(\frac{2}{3} + 4.2) \div (.125 \times \frac{1}{2})}{.375 \times (\frac{3}{5} - .16\frac{2}{3})}$.

17. $\frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} \times \frac{1 + \frac{1}{3}}{1 + \frac{1}{3}}$.

18. $\frac{.05 \times 1.02}{\frac{3}{4}} \div \left(\frac{1}{6\frac{1}{2}} - \frac{.005}{50}\right)$.

19. $\frac{3.7 + 1.05 + .508}{.43 - .005} \times (\frac{2}{3} - .5)$.

20. $(.0008 \div .008) \times 10000$.

21. $\frac{.468}{200} \times \frac{\frac{1}{8}}{2.12\frac{1}{2}} \div 5\frac{3}{5}$.

22. $\frac{1-.0001}{.5 \div 5} + 3\frac{1}{2}$.

23. $\frac{4-.002}{3 \div .03} + \frac{.04 \div .0002}{.01\frac{1}{2}}$.

24. $\frac{\sqrt{.4}+\sqrt{.9}}{\sqrt{.36}+\sqrt{.16}} + \frac{\sqrt{.4+.9}}{\sqrt{.36-.16}}$.

25. $\frac{\sqrt{\frac{4}{9}} \div \sqrt{\frac{16}{25}}}{\sqrt{\frac{1}{5}} \times \sqrt{5}}$.

26. $\sqrt{\frac{2}{3}} : \sqrt{\frac{3}{2}} :: \sqrt{7} : ?$

27. $\frac{\sqrt{4.9}+\sqrt{3.6}}{\sqrt{.25}-\sqrt{.01}}$.

28. $\frac{\sqrt{.0036}+\sqrt{490}}{\sqrt{.025}-\sqrt{.0025}}$.

29. $\sqrt{16} \times \sqrt{4}$, and $\sqrt{16} + \sqrt{4}$.

30. Is $\sqrt{16} \times \sqrt{4} = \sqrt{16 \times 4}$?

31. Is $\sqrt{16} + \sqrt{4} = \sqrt{16+4}$?

32. $\sqrt{\frac{3}{7}} \times \sqrt{\frac{7}{3}} \times \sqrt{\frac{2}{5}} \times \sqrt{\frac{5}{2}}$.

33. $\sqrt{\frac{3}{7}} + \sqrt{\frac{7}{3}} + \sqrt{\frac{2}{5}} + \sqrt{\frac{5}{2}}$.

34. $5\frac{1}{2} : 3\frac{1}{4} :: \frac{2}{3} : ?$

35. $\frac{2}{5} : 1\frac{1}{5} :: 8 : ?$

36. Why is $\frac{2}{3} : \frac{4}{5} :: \frac{15}{7} : ?$ the same as $\frac{3}{2} \times \frac{4}{5} \times \frac{15}{7} = ?$

37. $\sqrt{\frac{3}{4}} : \sqrt{\frac{3}{8}} :: 10 : ?$

38. $.05 : 3 :: 1.02 : ?$

39. $.4 : ? :: 6\frac{1}{2} : .03$.

40. $? : .001 :: .02 : .3$.

41. $4.0\frac{3}{4} : 1.00\frac{1}{2} :: .0\frac{2}{5} ?$

42. $\sqrt{2\frac{1}{2}} + \sqrt{4\frac{1}{5}}$

43. $\sqrt{2\frac{1}{2}} \times \sqrt{4\frac{1}{5}}$.

44. $\sqrt{16} \div \sqrt{4}$, and $\sqrt{16} - \sqrt{4}$.

45. Is $\sqrt{16} \div \sqrt{4} = \sqrt{16 \div 4}$?

46. Is $\sqrt{16} - \sqrt{4} = \sqrt{16-4}$?

47. $\sqrt{\frac{2}{5}} \div \sqrt{\frac{2}{5}}$.

48. $\sqrt{\frac{2}{5}} - \sqrt{\frac{2}{5}}$.

49. $\sqrt{1\frac{1}{2}} : \sqrt{2\frac{2}{3}} :: 6 : ?$

50. $1\frac{1}{2} : 2\frac{2}{3} :: 36 : ?$

What is the relation between the answers to the last two? Why?

51. $5 : \sqrt{.6} :: \sqrt{.15} : ?$

52. $\sqrt{.8} : 3 :: \sqrt{1.1} : ?$

53. $\sqrt{\frac{2}{3}} : \sqrt{\frac{9}{16}} :: \sqrt{\frac{4}{5}} : ?$

54. $\sqrt{\frac{2}{3}} \times \frac{9}{16} \times \frac{4}{5} = ?$

Are the answers to the last two alike? Why?

55. $\frac{3\frac{2}{5}+5\frac{3}{4}}{5\frac{1}{2}-.025}$.

56. $\sqrt{.002} : \sqrt{.004} :: \sqrt{7} : ?$

57. $\sqrt{10} : \sqrt{5} :: \sqrt{\frac{3}{5}} : ?$

58. $\sqrt{\frac{3}{4}} \times \sqrt{\frac{4}{3}}$.

59. $\sqrt{\frac{3}{4}} + \sqrt{\frac{4}{3}}$.

60. $\sqrt{\frac{.005 \div .01}{3-2.01}}$.

61. $\frac{(4\frac{1}{2} - 3\frac{1}{5})\ (.2 + 3)}{.3\ (5.6 \div .07)}$.

62. $\frac{(3 - \frac{1}{3}) \div (1 - \frac{2}{5})}{.04\ (1 - .05)}$.

63. $\frac{(4 \div .04)\ (4 - .04)}{(3 \times .03)\ (3 + .03)}$.

64. $\frac{2.08 \div (1 - \frac{1}{5})}{(2 + \frac{1}{5}) - (\frac{1}{3} \text{ of } \frac{3}{5})}$.

65. $\frac{1}{\frac{2}{3}} : ? :: \frac{\frac{3}{5}}{2\frac{1}{2}} : 4$.

66. $2 \times 3\frac{1}{5} : (5 - 1\frac{1}{6}) :: ? : \frac{1}{2} \text{ of } \frac{3}{7}$.

67. $(2 - \frac{3}{5}) \div \frac{4\frac{1}{2}}{\frac{3}{7} \text{ of } \frac{5}{9}}$.

68. $(1\frac{1}{2} - \frac{5}{8}) \div (.02 - .002)$.

69. $\frac{5\frac{1}{2} \times 2\frac{2}{3}}{2 \div .02} \div .125$.

70. $(\frac{2}{3})^2 : (1\frac{3}{5})^3 :: 8 : ?$

71. $(5\frac{1}{2})^2 \div (\sqrt{2.5} \times \sqrt{.016})$.

72. $(4\frac{2}{5})^3 \div (.12)^2 \times \sqrt{.4}$.

73. $(2.01)^2 + (5.3)^3 - (2\frac{1}{8})^2$.

74. $\left(5 \div \frac{2\frac{1}{4}}{\frac{1}{2} \text{ of } \frac{3}{8}}\right) \times \frac{13\frac{11}{12}}{4.2}$.

75. $\frac{\sqrt{3.6}\sqrt{-.16}}{(\frac{2}{3})^2 - \sqrt{\frac{1}{8}}} \div \sqrt{\frac{1\frac{1}{2}}{\frac{2}{3} \text{ of } \frac{5}{6}}}$.

76. $\frac{2}{3}$ of \$5 is what part of £4?

77. $\frac{1}{5}$ of $\frac{2}{3}$ of $1\frac{1}{4}$ yd. is what part of $2\frac{1}{5}$ m?

78. $\frac{1}{5}$ of $7\frac{1}{2}$ ft. is $\frac{2}{3}$ of how many meters?

79. $5\frac{1}{2}$ Kg is what part of 10 lb.?

80. $7\frac{1}{2}$ dg is what part of 10 gr.?

81. 4s. 6d. is what part of 2 Nap?

82. $5\frac{2}{3}M$. is what part of \$$2\frac{1}{4}$?

83. $14\frac{2}{7}$ less $\frac{\frac{1}{2} \text{ of } 8\frac{2}{5}}{14\frac{7}{10}}$ is $\frac{2}{3}$ of $\frac{7}{9}$ of what number?

84. Divide $7\frac{1}{3}$ into 3 parts, which shall be to each other as $1\frac{1}{3}$, $1\frac{1}{2}$, and 2.

85. Divide 12 into 3 parts, which shall be to each other as $\frac{2}{3}$, $\frac{3}{5}$, and 2.1.

86. Divide 9 into 4 parts, which shall be to each other as 2.5, 1.1, $\frac{1}{2}$, and $3\frac{1}{5}$.

87. Divide \$1250 into 3 parts, which shall be to each other as $\frac{2}{3}$ of 6, $\frac{1}{5}$ of 20, and $\frac{1}{7}$ of 14.

88. Divide \$3800 into 4 parts, which shall be to each other as 2×60, 3×50, 4×35, and 7×16.

89. Does it increase a proper fraction, or diminish it, to *extract* its root? Why?

90. Does it increase a proper fraction, or diminish it, to involve it to a power? Why?

91. Answer the last two questions with reference to an improper fraction?

92. $\frac{2}{5}$ of $3\frac{3}{4}$ liquid gallons is what part of .5 bu. ?

93. 2.6^{l} is what part of $\frac{2}{3}$ of 5 gal. ?

94. 4 mm is what part of .01 in. ?

95. 25^{Dl} is what part of 10 bu. ?

96. 3.2^{m} is what part of 1 rod?

97. A is 1.6^{m} in height, and B 5 ft. $10\frac{1}{2}$ in. What is the ratio of their statures?

98. What is the ratio of 3 ij to 4^{s}?

99. What is the difference between $\frac{3}{7}$ of $4\frac{1}{2} \times \frac{9\frac{3}{4}}{1\frac{3}{20}} \times \frac{1}{1^{1}5}$ of $\frac{7}{8}$ of £43 18s. $11\frac{1}{2}$d., and $3\frac{3}{8} \times \frac{1}{17\frac{1}{2}}$ of .56 of $1.75 \times 6\frac{1}{2}$ times $97.18?

100. Reduce $\frac{2}{3}$ of $4\frac{1}{2} \times 7\frac{1}{5} \times \frac{9}{19\frac{1}{2}}$ of $\frac{5}{8}$ of 3 oz. 4 dr. 2 scr. 5 gr. to the decimal of $\frac{6}{1^{1}}$ of .63 of $2\frac{37}{42} \times \frac{3}{1^{3}}$ of $6\frac{1}{2}$ times 7 lb. 3 oz. Av.

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469. ONE HUNDRED TEST EXERCISES IN APPLIED ARITHMETIC.

1. What per cent is 5 of 25? Of 10? Of 43?

2. Of what number is 11 6%? Of what 10%? Of what 7%?

3. I send an 8% note for $1000, dated July 1, 1876, to Buffalo for collection. The maker of the note, having gone into bankruptcy, pays only 75¢ on $1. The note is collected by my broker, April 30, 1877. What does he remit, charging $\frac{1}{2}$% for collecting?

4. The bank is discounting at 8%. I wish $200 for 60 da.

For what must the note be made? What for $350 for 30 da. at 10%? For $250 for 90 da. at 5%?

5. I send a 6% note for $500, dated May 10, 1875, with an indorsement July 1, 1876, of $150, to a broker in Chicago for collection. He collects it March 10, 1877, and charges me ¾% for collecting. What does he remit to me?

6. A real-estate broker received $2593.75 for the purchase of land. Reserving 3¾% commission on the purchase, what number of acres of land could be bought at $125 per acre?

7. If I buy bonds for 85 cents on a dollar which pay 3% semi-annual interest on their face, what per cent per annum does this give me on my investment, money being worth 10%?

8. What will be the duty in our currency on a case of silk mantillas, invoiced in Paris at 13950 francs, the rate of duty 60% *ad valorem?*

9. I invest $2000 in certain goods, which I sell at 50% advance, but at a cost of 3% on the sales for selling. Allowing 5% loss by selling on credit, what per cent do I make by the transaction?

10. How long does it take a principal to double at 6%? 7%? 8%? 10%?

11. How long does it take a principal to double itself at compound interest at 5%? At 10%? At 4%?

12. $5600. PHILADELPHIA, Jan. 11, 1871.

For value received, on demand, I promise to pay James Jones, or order, five thousand six hundred dollars, with interest, without defalcation. JOHN SMITH.

Indorsements: May 19, 1871, $500; Sept. 5, 1871, $200; Jan. 1, 1872, $300; April 17, 1872, $150.

What is due Jan. 11, 1873, by the United-States-Court *Rule?* What by the Merchant's Rule?

13. At what per cent will $240.80 amount to $325.08 in 5 yr. 10 mo.?

14. At what per cent will a given principal double in 12 yr.? In 15 yr.?

15. What principal will amount to $1617 in 3 yr. 6 mo. 15 da. at 8%?

16. Suppose an annual premium of $68.25 is paid for insuring a house worth $2275, what per cent is paid?

17. At a rate of $1\frac{1}{2}\%$ a year a warehouse is insured for $\frac{3}{4}$ of its value, paying thereon a premium of $202.50. What is the whole value of the warehouse?

18. A tax of $50,000 net is to be raised in a certain city on a valuation of $2,000,000. Supposing 3% to be uncollectible, and allowing 5% for collecting, what tax must be levied? What will be a man's tax who is assessed on $3500?

19. If the property in the city of Cleveland be assessed at $70,000,000, what must be the tax on each dollar to raise $294,000 for school purposes? At two-tenths of a mill on each dollar how much will be raised for the support of the library?

20. What per cent annual interest do I make by investing in railroad stock at 75, which pays 3% semi-annually, allowing 6% on the mid-year payment to the close of the year?

21. I bought 25000 feet of boards at $12.25 per thousand, and sold $\frac{1}{2}$ of them for what $\frac{2}{3}$ of the whole cost. What per cent did I gain on the part sold?

22. A merchant bought goods at 25% below their nominal price, and sold them at 20% above, thereby making $1920. How much did he invest?

23. I mark down 10% from the retail price goods which I was selling at 25% advance on cost. At what per cent advance on cost do I now propose to sell them?

24. In consequence of a rise of a certain article in the

market, I mark up 5% on my former retail price goods which I was selling at 20% advance on cost. At what per cent profit do I now propose to sell them?

25. A New-York bank in which I hold stock declares a 4% dividend. I draw a draft for the amount due me, and sell it at 1% premium in Omaha, receiving $707. How many shares do I own?

26. A bought 230 bales of cotton, each bale containing 450 lb., at 10¾ cents a pound, on a credit of 9 mo. He sold the cotton immediately for $12000 cash, and paid the present worth of the debt at 8%. What was his gain?

27. At what must cloth be bought to sell it at $9 per yd., and make 12½% profit?

28. A jeweller has a watch which cost him $150. He wishes to mark it so that he can fall 5% on the asking price, and still make 20%. How must he mark it?

29. A bookseller sells a book for $1.20, and makes 25% thereby. What would he have made had he sold it at $1.28?

30. When ⅔ the selling price equals the cost, what per cent is made? When ½ the selling price equals the cost? When the selling price is ⅝ of the cost, what per cent is lost?

31. How much water must be added to 1 gal. pure alcohol to make a mixture 75% alcohol? How much, to make one 50%? 40%?

32. A railroad has been constructed through a farm, making it necessary to build fences at a cost of $750, which must be renewed every 15 years. What should the owner receive to meet this expenditure, at 6% compound interest?

33. If a square orchard contains 2916 trees, how many are in a row on each side?

34. A man has a rectangular board 128 in. long and 32 in. wide, from which he makes a square table as large as possible. Required its length, no allowance being made for *sawing*.

35. What would it cost to enclose a square lot containing 160 acres, with a fence costing at the rate of $4 per rod?

36. What cost 3 piles of 4 foot wood, one 58 ft. long and 5 ft. high, another 70 ft. long and 5½ ft. high, and the other 65 ft. long and 6 ft. high, at $5.50 per cord?

37. I have a cylindrical cistern 6 ft. deep and 6½ ft. in diameter. How much shall I increase its capacity if I increase each of its dimensions 25%? 50%? 100%?

38. How much is a rectangular bin increased in capacity by increasing two of its dimensions 10%? By increasing all three of its dimensions 10%? If I double two of its dimensions? If I double all three dimensions?

39. Suppose a rectangular field to be three-eighths of a mile in length, and one-fourth of a mile in width. What distance will be saved by walking directly from any corner to the corner diagonally opposite, instead of going by the line of the fence from the one point to the other?

40. A ladder 30 ft. in length was found to reach just to the eaves of a building when its foot was 12 ft. from the foundation. What was the height of the building?

41. The eaves of a house are at the same height, and 30 feet apart. The ridge-pole is 12 feet higher than the eaves, and just midway between them. The house is 40 feet long. How many shingles will it take to cover the roof, if each shingle covers a space 6 inches long and 4 inches broad?

42. A third-pitch "square roof" is to be put on a rectangular house, 36 by 42 ft., with a flat deck at top 8 ft. above the plates. What will be the size of the deck, what the length of the side rafters, and what the 4 corner rafters? (See p. 334, Ex. 7.)

43. How many revolutions in a minute does a 6 foot drive-wheel of an engine make when the engine is running at the rate of 30 mi. an hour?

44. If a ball 1 foot in diameter weighs 100 lb., what is the weight of one 3 feet in diameter, made of the same material?

45. How large a square can be cut from a circle 36 in. in diameter?

46. How large a cube can be cut from a sphere 2 ft. in diameter?

47. If a solid globe of 4 in. diameter weigh 20 lb., what will one of the same material 6 in. in diameter weigh?

48. How many boxes of common double tin, 100 12×17 in. sheets in a box, will it take to cover a hemispherical dome of 20 ft. diameter, allowing $\frac{1}{2}$ in. lap on end and side of each sheet?

49. What is the area of a trapezoid whose parallel sides are 30 and 50 rods, and whose altitude is 40 rods?

50. What is the diameter of a sphere whose surface is 100 sq. ft.?

51. What is the diameter of a sphere whose volume is 150 cu. ft.

52. The sun's diameter being 852573 mi., and the earth's radius 3958 mi., how many times as large as the earth is the sun?

53. The diameter of the moon being to that of the earth as 3 : 11, what is the relation between their volumes?

54. What is the diameter of a grindstone when it is $\frac{1}{3}$ worn away, its original diameter having been 2 ft.? What when $\frac{2}{3}$ worn? When $\frac{1}{2}$ worn? When $\frac{1}{4}$? $\frac{3}{4}$?

55. If a ball of thread is 4 inches in diameter, what will be the diameter in each of three conditions, — when $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of it are wound off? What part of the thread will be left when the diameter is reduced to 2 inches?

56. When Gen. Tom Thumb (Charles S. Stratton) was 5 years old, he measured 2 ft. in height, and weighed 16 lb. What would be the weight of a man of similar form who was 6 ft. tall?

57. What are the dimensions of a rectangular box *containing 3000* cubic feet, the dimensions being to each other *as 2, 3, and 4?*

58. How many yards of carpeting ¾ yd. wide must I buy to carpet a room 20 by 25 feet, the strips running lengthwise of the room, and there being 4 inches waste on each strip in matching?

59. I sell 12 logs at $10 per M, board measure. Six logs are 12 ft. long, and six 14 ft. The first scale 28 in., 30 in., 40 in., and three 32 in.; the others, two 30 in., three 35 in., and one 20 in. What do the logs bring me, reckoning by *Scribner's* "Log Book"?

60. How many cubic feet of hewn timber in 3 logs measuring 15 ft. long, 21 and 25 in. in diameter; 20 ft. long, 26 and 30 in. in diameter; and 32 ft. long, 30 and 36 in. in diameter, measured by *Scribner's* rule?

61. How many minutes will there be in the month of February, 1880?

62. A lady bought 6 silver spoons, each weighing 3 oz. 3 pwt. 8 gr., at $2.25 an ounce, and a gold chain weighing 14 pwt., at $1.25 a pwt. What was the cost of both spoons and chain?

63. What is the difference between 32 liters and a bushel? What is the difference between a barrel and 1¼ hektoliters?

64. How many gallons does a tub 18 in. deep contain, whose top is 16 in. in diameter, and bottom 20 in.?

65. If telegraph-poles are 66 ft. apart, and a train passes one every 3 sec., what is the rate in miles per hour?

66. A physician having 1 ℔ ℥ iij ʒ iv ℈ ij gr. xij of a certain medicine, put it up in gr. xx packages. How many did it make?

67. The United-States "Trade Dollar" (silver) weighs 420 gr., and the common half-dollar 12.5^{g}. How much more is the trade dollar actually worth than two common half-dollars?

68. How many bushels in a bin 3.2^{m} long, 1^{m} wide, and 2^{m} deep. How many hektōls? How many kilōgs of wheat will it contain at 60 lb. to the bushel?

69. Cleveland, O., is in longitude 81° 47′ W., and Boston in 71° 4′ 9″ W. When it is 5 A.M. at Boston, what time is it at Cleveland?

70. What is the difference in longitude between two places whose difference in time is 52 min. 18 sec.?

71. In crossing the State of Michigan from Detroit to South Haven, which are nearly on the same parallel, I find my watch, which is set to Detroit time, 12½ min. fast at South Haven. Now 51 mi. make a degree of longitude on this parallel. What is the width of the State at this point?

72. The specific gravity of milk is 1.032. What does 1^{Hl} weigh?

73. A pail containing 1^{Hl} of cider is filled, and the cider found to weigh 101.8^{Kg}. What is the specific gravity of cider?

74. One millil of sulphuric acid is found to weigh 18.42 dg. What is its specific gravity?

75. The specific gravity of linseed-oil is .94. How much would a cask of 2^{Hl} weigh?

76. I find that a liter of alcohol weighs 8^{Hg}. What is its specific gravity?

77. The specific gravity of cast iron being 7.25, what is the weight of a 12 in. cast-iron shell (hollow sphere), the shell being 1½ in. thick?

78. The specific gravity of iron being 7.25, what is the weight of a 10 in. cast-iron cannon-ball?

79. The specific gravity of common loose earth is about 1.5. A cubic yard makes a good-sized load for a span of horses. What does it weigh?

80. The specific gravity of common rocks is about 2.5. What is the weight of the earth if its material averages the same?

81. Required the time of day, provided the time past noon *equals* ⅔ of the time to midnight.

82. A quantity of flour lasts a man and wife 9 days, and the wife alone 27 days. How long would it last the man alone?

83. If a 2 in. pipe will fill a cistern in 6 hours, how long will it take a 3 in. pipe to fill it, the water flowing at the same velocity?

84. A agreed to labor for $2.50 per day, on condition that he should forfeit 50¢ every day he was idle. At the end of 100 days he received $190. How many days was he idle?

85. A and B together can do a piece of work in 6 days; but B alone requires 10 days to do it. In what time can A do it alone?

86. Three men, A, B, and C, agree to do a certain piece of work. A and B can do the work in $6\frac{2}{3}$ days, B and C in 12 days, and A and C in 10 days. How long will it take each separately to do it?

87. In a mixture of gold and silver consisting of 100 oz. there are 6 oz. of silver. How much gold must be added that there may be $\frac{2}{5}$ oz. of silver to 10 oz. of gold?

88. A man bought a bar of gold at $192 per lb. and sold it for $16 per oz., weighing it in both cases by avoirdupois weight. How much did he gain, the true weight of the bar being 5 pounds? Did he gain, or lose, by selling by avoirdupois instead of troy?

89. French coin is of the same fineness as our own. A franc is 19.3 cents. Our gold dollar weighs 25.8 gr., and our silver dollar $412\frac{1}{2}$ gr. What, then, should be the weight of a silver franc in grams? What of a Napoleon?

90. When the temperature is 80° F., what is it C.? Blood-heat is 98° F.: what is it C.? —28° F. is what C.?

91. 57° C. is what F.? —18° C. is what F.?

92. What day of the week was the 4th of July, 1776? What day of the week will it be in 2000?

93. On what day of the week were you born?

94. On what day of the week will Christmas come this year? What the 4th of July?

95. If 20 yards of cloth, 1 yard wide, shrink 4% in length and 5% in breadth by sponging, what will be the loss in square yards?

96. At the breaking out of the late Franco-Prussian war the German government made a 5% war loan, which was taken at 88; and the French a 3%, which was taken at 65½. Which paid the higher rate of interest, assuming that both ran indefinitely? Which, if the loans ran only 10 years?

97. Calling the diameter of the earth 7912 miles, and the height of the highest mountain 28000 feet, what elevation would represent the mountain on an artificial globe 2 feet in diameter?

98. Bought cranberries at $4.80 per bushel, and sold them at 15¢ per qt., measuring them out by liquid measure, whereas I bought them by dry measure. What per cent was my profit?

99. Compare the amount of a $500 10% note running 4 yr. at simple interest, annual deferred int. and compound int.

100. How much must I invest in United-States 4% bonds at 102⅞ to give me a quarterly income of $500?

APPENDIX.

ERATOSTHENES' SIEVE.

1. To find the prime numbers between any given limits. — *Write down all the odd numbers, 1, 3, 5, 7, 9, etc. Over every third from 3 write 3; over every fifth from 5 write 5; over every seventh from 7 write 7; over every eleventh from 11 write 11; and so on. Then all the numbers which are thus marked are composite, and the others, together with 2, are prime.* (Why?)

Also, the figures thus placed over are the factors of the numbers over which they stand. (Why?)

Ex. — Find all the prime numbers less than 100.

				3			3.5	
1	3	5	7	9	11	13	15	17
	3.7		5	3			3.11	5.7
19	21	23	25	27	29	31	33	35
	3.13			3.5		7	3.17	
37	39	41	43	45	47	49	51	53
5.11	3.19			3.7	5.13		3.23	
55	57	59	61	63	65	67	69	71
	3.5	7.11		3		5.17	3.29	
73	75	77	79	81	83	85	87	89
7.13	3.31	5.19		3.11				
91	93	95	97	99				

Hence, rejecting all the numbers which have *superiors*, the primes less than 100 are 1, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, together with the number 2.

This process may be extended indefinitely, and is the method by which primes are found even by modern computers. It was invented by Eratosthenes, a learned librarian at Alexandria (born B.C. 275). He inscribed the series of odd numbers upon parchment, then, cutting out such numbers as he found to be composite, his parchment with its holes somewhat resembled a *sieve:* hence this method is called "*Eratosthenes' Sieve.*"

GENERAL METHOD OF FINDING THE GREATEST COMMON DIVISOR.

This rule is based on the two following principles:—

2. PROP. 1.—*A divisor of a number is a divisor of any number of times that number.*

This is self-evident, since, if a is contained in b c times, it is contained in $2b$ twice as many times, in $3b$ 3 times as many times, etc.

3. PROP. 2.—*A divisor of any two numbers is a divisor of their sum, and also of their difference.*

This is also self-evident. Thus, if a is contained in m 5 times, and in n 3 times, it is evident that it is contained in their sum 8 times, and in their difference 2 times.

4. Rule.—*To find the G. C. D. of two numbers, divide the greater by the less, and this divisor by the remainder, continuing to divide the last divisor by the last remainder until there is no remainder. The last divisor is the G. C. D. sought.*

DEMONSTRATION.—In order to demonstrate this rule, let us find the G. C. D. of 42 and 138. Performing the operation according to the rule, as in the margin, we are to prove that 6 is the G. C. D. of 42 and 138.

```
42 ) 138 ( 3
     126
     ---
      12 ) 42 ( 3
           36
           --
            6 ) 12 ( 2
                12
                --
```

As 42 is its own G. D., if it divides 138 it is the G. C. D. sought. Trying it, we find a remainder 12. Now any divisor of 42 is a divisor of 3 times 42, or 126 (PROP. 1), and any divisor of 126 and 138 is a divisor of their difference 12 (PROP. 2). Hence the G. C. D. sought cannot be greater than 12. Moreover, any number which divides 12 and 42 divides 138, which is the sum of 12 and 3 times 42 (PROPS. 1 and 2). Thus the question is reduced to finding the G. C. D. of 12 and 42.

In like manner we can reduce it to the question of finding the G. C. D. of 6 and 12. But this is 6. Hence 6 is the G. C. D. of 42 and 138.

Ex. 1.—Find the G. C. D. of 9131 and 13133.

An arrangement like that in the margin will be found convenient in performing the divisions. Placing the first divisor, the smaller number, on the right, divide and write the quotient at the right of both. Thus 9131 is contained in 13133, 1 time, with 4002. Now, using this remainder as divisor, we continue the work as in the margin.

OPERATION.

13133	9131	1
9131	8004	2
4002	1127	3
3381	621	1
621	506	1
506	460	4
115	46	2
92	46	2
23		

2. Find the G. C. D. of 4420, 3094, and 1326. Of 1445, 1190, and 204.

The common method of solving such problems is to find the G. C. D. of the two least numbers, and then of this G. C. D. and the next larger of the numbers, etc. But familiarity with the principles on which the operations are based will suggest better ways. Thus, for the above, we have the following: —

4420	3094	1326	2
2210	1547	663	2, 3
2210	1326	663	10

221
2
442 = G. C. D.

1445	1190	204	5, 8
1360	1020	170	2
85	170	34	2, 5, 2
68	170	34	

17 = G. C. D.

In the *first* of these we reserve the evident common factor 2 as a factor in the G. C. D. sought. Hence we have to find the G. C. D. of 663, 1547, and 2210. Dividing 1547 by 663, we reduce the problem to finding the G. C. D. of 663, 221, and 2210. Then, as 221 is found to be the G. C. D. of these, $221 \times 2 = 442 =$ G. C. D. sought.

In the second, the problem is first reduced to finding the G. C. D. of 204, 170, 85. Then, as 85 is found to be a divisor of 170, it becomes a question between 85 and 204. The order of operation in this case is, $1190 \div 204$; $1445 \div 170$; $170 \div 85$; $204 \div 85$; $85 \div 34$; $34 \div 17$.

The student will have no difficulty in applying this method of finding the G. C. D. of several numbers, if he is careful to mark at each step what numbers are now to be examined, and always divides first by the least number, proceeding in order to the greatest under comparison.

LEAST COMMON MULTIPLE.

5. Rule. — I. *Write the numbers in a horizontal line, and divide by any prime number that will divide two or more of them without a remainder, placing the quotients and numbers undivided in a line below.*

II. *Divide this line as before, and thus proceed till no two numbers are divisible by any number greater than 1. The continued product of the divisors and numbers in the last line will be the L. C. M. of the numbers.*

Ex. 1. — Find the L. C. M. of 45, 81, 96, and 35.

Applying the rule to the solution of this example, we have the work in the margin. The principle is the same as given in the text.

5) 45 81 96 35
3) 9 81 96 7
3) 3 27 32 7
1 9 32 7

5.3.3.9.32.7 = L. C. M.

2. Find the L. C. M. of 54479 and 35741.

In this case it is not easy to discern a common factor, if the numbers have one. We may therefore apply the method for finding the G. C. D. Having found the G. C. D., we can divide the smaller number by it, and find the factor by which the larger number is to be multiplied, in order to give a product which will contain the smaller.

3. Find the L. C. M. of 31861, 88409, and 63269.

PROGRESSIONS.

6. An Arithmetical Progression is a series of numbers which increase or decrease by a common difference, as 3, 5, 7, 9, 11; or 28, 23, 18, 13, 8.

7. The Last Term of an *increasing* arithmetical progression is evidently equal to the first term + the common difference taken as many times as there are terms less 1. Thus the 5th term of the 1st series above is 3 + 4 times 2 = 11.

The last term of a *decreasing* arithmetical progression is equal to the first term — the common difference multiplied by the number of terms less 1. Thus the 5th term of the 2d series above is 28 — 4 times 5 = 8.

8. The Sum of an arithmetical progression is ½ the sum of the extremes multiplied by the number of terms.

This will be evident from an inspection of this operation. Hence the $Sum = \left(\frac{3+11}{2}\right) \times 5 = 35$, the Sum of the Series.

$$\begin{aligned} 3 + 5 + 7 + 9 + 11 &= \text{sum.} \\ 11 + 9 + 7 + 5 + 3 &= \text{sum.} \\ \hline 14 + 14 + 14 + 14 + 14 &= \textit{twice} \text{ the sum.} \end{aligned}$$

Ex. 1. — First term 7, common difference 4, series increasing, find the 10th term and the sum.

2. First term 134, common difference 7, series decreasing, find the 8th term and the sum.

3. A 10% note for $300, bearing annual interest, has been running 8 yr., and no interest has been paid. What is due, allowing simple interest on the deferred payments of annual interest?

The 8th year's interest is $30, the 7th $33, the 6th $36, etc. Hence the interest is an arithmetical progression of 8 terms, of which 30 is the first, and 3 the com. diff. The last term is therefore 51, and the sum $\left(\frac{51+30}{2}\right) \times 8 = 324$. Amount of note, $624.

9. A Geometrical Progression is a series of numbers which increase or decrease by a common multiplier, called the *rate*. If the rate is more than 1, the series is increasing; if less than 1, it is decreasing. Thus 3, 9, 27, 81, 243, is an increasing geometrical progression, rate 3. 6561, 729, 81, 9, is a decreasing geometrical progression, rate $\frac{1}{9}$.

10. The Last Term of a *geometrical progression* is the first multiplied by the rate raised to a power whose index is 1 less than the number of terms. This appears when we consider that the 2d term is the first multiplied by the rate, the 3d is the first multiplied 2 times in succession by the rate, etc.

11. The Sum of a *geometrical progression* is the difference between the last term multiplied by the rate and the first term, divided by the *rate* — 1 if the series is *increasing*, and by 1 — *the rate* if it is decreasing.

Thus, taking the series 3, 9, 27, 81, 243, of which the rate is 3, the sum is $\frac{3 \times 243 - 3}{3 - 1}$, or $\frac{729 - 3}{2} = 363$. An inspection of the following will indicate the reason for the rule —

$$\begin{array}{rrl} 729 + 243 + 81 + 27 + 9 & & = 3 \text{ times the sum.} \\ 243 + 81 + 27 + 9 + 3 & & = \text{the sum.} \\ \hline 729 & - \quad 3 & = (3 - 1) \text{ times the sum.} \end{array}$$

Again,

$$\begin{array}{rrl} 6561 + 729 + 81 + 9 & & = \text{the sum.} \\ 729 + 81 + 9 + 1 & & = \frac{1}{9} \text{ the sum.} \\ \hline 6561 & - \quad 1 & = (1 - \frac{1}{9}) \text{ times the sum.} \end{array}$$

Ex. 1. — First term of a geometrical progression 7, rate 4. What is the 8th term? What the sum?

2. First term 6250, rate $\frac{1}{5}$. What is the 6th term? What the sum?

DIVISION OF UNITED-STATES PUBLIC LANDS.

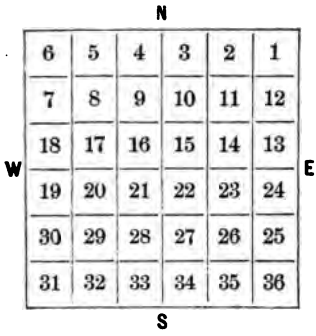
12. When a new territory is to be surveyed, the first thing the surveyor does is to run one or more north and south lines through some convenient parts of it. These are run with great care, are carefully marked by posts, stones, marks upon trees, or other means, throughout their entire length, and are called PRINCIPAL MERIDIANS. In a similar way one or more east and west lines, called BASE LINES, are run and marked. After this the whole country is checked up into townships by running north and south lines parallel to the principal meridian, and six miles apart and east and west lines in a similar manner parallel to the base line. The north and south rows of these townships are called *Ranges*, and are numbered east and west from the *principal meridian*. The townships in each row are numbered north and south from the *base line*. Thus in Ohio the western boundary of Pennsylvania is the eastern principal meridian from which ranges are numbered up to Range XX. West, which reaches the western boundary of Huron County. Again: the western boundary of Ohio is another principal meridian from which ranges are numbered eastward to Range XVII. East. The base line in this State runs along the southern boundary of Paulding, Seneca, and Huron Counties. From this line townships are numbered, as Town 1 North, Town 2 North, etc.; Town 1 South, Town 2 South, etc. Townships are designated thus: T. 2 N., R. 4 E.; T. 8 S., R. 5 W., etc. The first of these is read, "Town 2 North, Range 4 East," and locates Brown Township in Paulding County, 6 miles north of the base line, and 18 miles east of the west meridian, or west line of the State.

The base line in Michigan runs along the north line of the second tier of counties; viz., Wayne, Washtenaw, etc. The principal meridian commences at the Ohio line, and runs north between Lenawee and Hillsdale Counties. The *names* which we apply to the townships form no part of a description of them as given in deeds: such description is by range and number. Sometimes (usually) the *political* divisions we call townships correspond to these surveyed and recorded towns: but often other divisions are made for political purposes; otherwise all the townships would be regular (except those along lake and river boundaries, etc.).

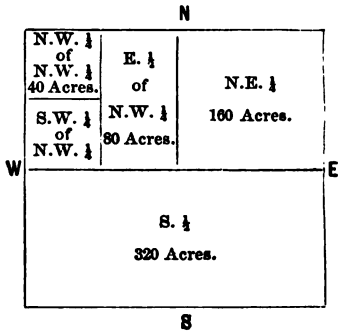
The following cuts illustrate the division of a township into sec-

tions and the subdivisions of sections. These sections are designated by number, beginning at the north-east corner, and numbering

A TOWNSHIP.



A SECTION.



across to the west, then back to the east, and thus back and forth. The division of the section is not made in the original survey; but the middle of the sides on the boundary-lines are marked so that all that is afterward necessary is to run the lines across in order to divide the sections into quarters. For purposes of sale these quarters are again divided into halves and quarters. The following designations will be readily understood: "North-west quarter of the north-west quarter of Section 10," written, "N.W. ¼ of N.W. ¼ of Sec. 10." See cut of section. So the east half of the north-west quarter is designated, "E. ½ of N.W. ¼ of Sec. 10."

Descriptions may be written on the board, and the pupils required to illustrate the location by diagrams on their slates.

N.E. ¼ of S.E. ¼, Sec. 20, T. 7 N., R. 9 E.
W. ½, S.W. ¼, Sec. 17, T. 10 S., R. 6 W.
S. ½, Sec. 28, T. 1 S., R. 15 E.
E. ½, N.E. ¼, Sec. 8, T. 6 N., R. 18 W.
W. ½, Sec. 16, T. 13 N., R. 11 E.
E. ½, S.E. ¼, Sec. 32, T. 5 S., R. 13 E.

THE METRIC SYSTEM.

13. The *Metric System*, originally devised and adopted by the French, makes *The Meter* the fundamental unit. It was designed that the meter should be $\frac{1}{10000000}$ part of a quadrant of a meridian of the earth. With this design an arc of the meridian, starting from the parallel of Dunkirk in the extreme north of France, and running the entire length of France, and terminating in the parallel of Barcelona in the north of Spain, was measured by Delambre and Méchain, as directed by the French Government. From this measurement the whole quadrant was computed, and the meter established as $\frac{1}{10000000}$ part of it. It is now known that there are irregularities in the form of the earth which would make such measurements give different results when taken in different places, and that the meter thus established is about $\frac{1}{5000}$ of an inch too short.

The meter being thus established, the *liter* is made a cubic *decim*, and this amount of pure water at the temperature of melting ice is made the kilog.

The Metric System has now come to be adopted by most civilized nations, although generally only permissively, as a system legally recognized, but which may be used by the people, or not, as they see fit. Nevertheless, all nations, except the French (and they to a considerable extent), continue to use their various and older systems. It is, however, coming to be pretty generally accepted for scientific and philosophical purposes, and its cosmopolitan character makes it specially desirable that it should be understood by all who lay any claim to general intelligence. That the Linear Measures, Measures of Weight and of Capacity, will soon be in general use in this country, scarcely admits of a doubt; and hence the subject demands a place in our schools.

In attempting to teach the metric system, it is of the first importance that the pupils be made familiar with the measures themselves. *The Metric Bureau*, Boston, is organized for the purpose of furnishing apparatus for teaching, and information upon this subject.

14. Value of Foreign Coins in U. S. Money (Gold) as proclaimed by the Secretary of the Treasury, Jan. I, 1878.

COUNTRY.	UNIT.	METAL.	U. S.
Argentine Republic[1]	Peso fuerte	G.	$1.00
Austria	Florin	S.	.45,3
Belgium	Franc	G. & S.	.19,3
Bolivia	Dollar	G. & S.	.96,5
Brazil	Milreis of 1000 reis	G.	.54,5
Bogota	Peso	G.	.91,2
Canada[1]	Dollar	G.	1.00
Central America	Dollar	S.	.91,8
Chili	Peso	G.	.91,2
Cuba[1]	Peso	G.	.92,5
Denmark	Crown	G.	.26,8
Ecuador	Dollar	S.	.91,8
Egypt	Pound of 100 piasters	G.	4.97,4
France	Franc	G. & S.	.19,3
Great Britain	Pound sterling	G.	4.86,6½
Greece	Drachma	G. & S.	.19,3
German Empire	Mark	G.	.23,8
Hayti	Dollar	S.	.95,2
India	Rupee of 16 annas	S.	.43,6
Italy	Lira	G. & S.	.19,3
Japan	Yen	G.	.99,7
Liberia	Dollar	G.	1.00
Mexico	Dollar	S.	.99,8
Netherlands	Florin	S.	.38,5
Norway	Crown	G.	.26,8
Paraguay[1]	Peso	G.	1.00
Peru	Dollar	S.	.91,8
Porto Rico	Peso	G.	.92,5
Portugal	Milreis of 1,000 reis	G.	1.08
Russia	Ruble of 100 copecks	S.	.73,4
Sandwich Islands	Dollar	G.	1.00
Spain	Peseta of 100 centimes	G. & S.	.19,3
Sweden	Crown	G.	.26,8
Switzerland	Franc	G. & S.	.19,3
Tripoli	Mahbub of 20 piasters	S.	.82,9
Tunis	Piaster of 16 caroubs	S.	.11,8
Turkey	Piaster	G.	.04,3
U. S. of Colombia	Peso	S.	.91,8
Uruguay[1]	Patacon	G.	.94,9

[1] Taken from the Treasury circular for 1875, as they are not mentioned in 1878.

PRESENT UNITED STATES COINAGE.

412½ gr.

SILVER.

192.9 gr.

SILVER.

BRONZE.

96.45 gr.

SILVER.

36.58 gr.

SILVER.

5 grains

NICKEL.

NICKEL.

PRESENT GOLD COINAGE

OF THE UNITED STATES.

516 gr.

258 gr.

The Eagle—Ten Dollars. 1870.

Half-Eagle—Five Dollars. 1870.

25.8 gr.

UNITED STATES COINS IN USE BUT NOT NOW COINED.

SILVER.

SILVER.

SILVER.

GERMAN COINS.

20 Mark
$5.70

GOLD.

28.5¢

SILVER.
373

ENGLISH COINS.

Sovereign—$4.8665

GOLD.

24-3¢

SILVER.

SILVER.

COPPER.

COPPER.

FRENCH COINS.

Napoleon—$3,86

GOLD.

5 Centimes—4.8¢.

BRONZE.

19-3¢.

SILVER.

Page 153. *2–5*, 2224.9083, 1516.002, 1674.986, $1083.395.

Page 154. *5–20.* 921.984, 677.584, 47.454, .997, 4.553, 44.744, 7.76, .4563, 37.666, 9.9, .999, .85, 4.51, *, $211.09, 24.05.

Pages 155, 156. *1–11.* *, 32.9, $10.26, *, 12.12½, *, I owed $2.32, 21.25, 82, 5⅛ *yd.*, $6089.25.

(It is deemed inexpedient to give more answers than are found in the text, to the examples in the fundamental operations in Decimal Fractions. To give the answers to such problems is to tell the student exactly the thing—the position of the decimal point—which he needs to learn by solving the example.)

Pages 161, 162. *2.* $97.88. *3.* $2.98. *4.* 1½ doz. *5.* $9.37½. *6.* 12.2+ *hr.* *7.* 20 *mi.* *8.* $346.50. *9.* .53+ of a day. *10.* 40.8 *rd.*

Page 163. *2.* $7.84. *3.* $7.18. *4.* $15.49.

Pages 164-166. *2.* $489.834. *3.* $197.041. *4.* $13000. *5.* $57.04. *6.* $41.56+. *7.* $294.50. *8.* $11.72—. *9.* $78.83. *10.* $9.92. *11.* $11⅓. *12.* 1725. *13.* $58.12½. *14.* $15500. *15.* $7.50. *16.* $8.03—. *17.* 22½ *c.* *18.* 9⅓ *c.* *19.* To pay freight. *20.* $84.37½. *21.* 1½ *c.*+. *22.* $5600. *23.* It will cost $32.50 more by rail. *24.* 15 *c.*—. *25.* $186.60. *26.* $1071.04. *27.* No diff. *28.* 192. *29.* 2.461+.

Pages 170, 171. *10.* $193000, $4.90—, $3.04—, $49.91—, $386, 2849^{cr}.74. *14.* 420.17—. *15.* 105+. *16.* 4.20+. *17.* $41.89+. *18.* $613.56—, $193.94—.

Page 179. *3.* 102.85+ *bu.*, 30.4— *bbl.* *4.* 3.4—. *5.* 2¼—. *7.* $\frac{5}{6}\frac{7}{7}\frac{7}{2}\frac{5}{0}$ (about $\frac{3}{7}$), 37.2+.

Pages 181, 182. *2.* Gold is 127½ *gr.* heavier. *4.* 7.142+ *bbls.* Salt, 10.204+ *bbls.* Flour. *6.* 53⅓. *8.* 440.811+. *10.* 3 *pwt.* *11.* $65. *12.* 28. *13.* $0.314+. *14.* 3446.

Pages 193, 194. *3.* £601 8*s.* 9*d.* 2 *fr.* *4.* 87 *lb.* ℥ ij. *5.* 41 *rd.* 1 *yd.* 1 *ft.* *6.* 704 *bu.* 6 *qt.* *7.* £121 4*s.* 10*d.* 3 *fr.* *8.* 19 *lb.* 2 *oz.* 4 *pwt.* 23 *gr.* *9.* 931 *cd.* 9 *cu. ft.* 410 *cu. in.* *10.* 598 *sq. yd.* 5 *sq. ft.* 119 *sq. in.* *11.* 18 *bbl.* 24 *gal.* 3 *qt.* ¼ *pt.* *12.* 9.92875 *bu.* *13.* 33.168+ *rd.* *14.* 2 sections.

Page 196. *3.* £4 4*s.* 2*d.* *5.* 6 *bu.* 2 *pk.* 2 *qt.* *6.* 2 *pk.* 7 *qt.* 1 *pt.* *7.* 1 *pk.* 4 *qt.* *8.* 3 *bbl.* 28 *gal.* 1 *qt.* *9.* 2 *lb.* 3 *oz.* 11 *pwt.* 7 *gr.* *10.* 11 oz. 4 *pwt.* 4 *gr.* *11.* 3 *mi.* 69 *rd.* 13 *ft.* 6 *in.* *12.* 26.3109375 *A.* *13.* 1.56 *cd.* *14.* 3⅛ *yd.* *15.* 17 *cwt.* 80 *lb.* *16.* 268 *A.* 115 *sq. rd.* *17.* ℥ iij ʒ v ℈ ij. *18.* 20 *gal.* 2 *qt.* 1 *pt.* *19.* 3 *oz.* 20. 2 *in.*

Page 197. *2.* 5711. *3.* 174. *4.* 96. *5.* 269. *6.* 108. *7.* 58. *8.* 69. *9.* 87. *10.* 86.

Page 198. *2.* 21 *yr.* 8 *mo.* 19 *da.* *3.* 19 *yr.* 4 *mo.* 21 *da.* *4.* 254 *yr.* 0 *mo.* 27 *da.* *5.* 1 *yr.* 2 *mo.* 10 *da.* *6.* 2 *yr.* 2 *mo.* 20 *da.* *7.* 3 *yr.* 7 *mo.* 21 *da.* *8.* 3 *yr.* 6 *mo.* 3 *da.*

Page 199. *2.* 47 *yr.* 7 *mo.* 16 *da.* *4.* 7 *yr.* 26 *da.* *5.* Nov. 18, 1862. *7.* Dec. 28, 1865. *8.* 824 *da.*

Pages 200, 201. *3.* £44 16*s.* 6*d.* *4.* 18 *cd.* 96 *cu. ft.* *5.* 90 *gal.* 1 *pt.*, 1287 *gal.* 2 *qt.* *6.* 6 *lb.* 2 *oz.* 6 *pwt.*, 51 *lb.* 19 *pwt.* 12 *gr.* *7.* 396 *A.* 8 *sq. rd.* 14 *sq. yd.* 36 *sq. in.* *8.* 552 *mi.* 20 *rd.* 14 *ft.*, 1086 *mi.* 237 *rd.* 9 *ft.* 6 *in.* *9.* 1921 *da.* 6 *hr.* 46 *min.* 40 *sec.* *10.* 459° 10′ 30″, 61° 13′ 24″, 132° 39′ 2″, 102° 2′ 20″. *11.* 14 *lb.* ℥ ix ʒ j ℈j, 44 *lb.* ℥ iij ʒ iv, 88 *lb.* ℥ vij.

Page 202. *4.* £2 16*s.* 8$\frac{8}{9}$*d.* *5.* 5 *lb.* 12$\frac{2}{5}$ *oz.* *6.* ℥ vij ʒ ij *gr.* 6$\frac{3}{8}$. *7.* 2 *A.* 10 *sq. rd.*, 122$\frac{3}{8}$ *sq. rd.*, 1 *A.* 93$\frac{1}{18}$ *sq. rd.* *8.* 16 *rd.* 11 *ft.* 5$\frac{1}{16}$ *in.* *9.* 1 *gal.* 1 *qt.* 1$\frac{5}{11}$ *pt.* *10.* 2$\frac{3}{7}$ *qt.* *11.* 2 *cd.* 74$\frac{4}{5}$ *cu. ft.* *12.* 1 *yd.* 1 *ft.* 2 *in.* *13.* 5$\frac{1}{16}$. *14.* Last quotient 16.08+.

Pages 203-206. *1.* 8 A.M., 1 P.M. *2.* Last 7:14 P.M. *3.* 2 *min.* 58.4 *sec.* *4.* 7$\frac{1}{2}$°. *5.* 52$\frac{1}{2}$°, 37$\frac{1}{2}$′, 30° 50′. *6.* 56 *min.* P.M. and 4 *min.* P.M. *7.* 12 *m.* 42.5 *sec.*, 5 *hr.* 12 *min.* 42.5 *sec.* P.M., 7 *hr.* 47 *m.* 17.5 *sec.* P.M. *8.* 20° 38′. *9.* 90° 15′ 15″. *10.* 2° 45′. *11.* 651$\frac{1}{2}$+ *mi.* *12.* 10 *hr.* 22 *m.* 8 *sec.* P.M. *13.* 31 *m.* 36 *sec.* *14.* 52.2+. *15.* 189.375 *mi.*, 252.5 *mi.*, 378.75 *mi.*, 170.4375 *mi.* *16.* It will be 19 *m.* slow, 87° 43′. *17.* 4 *hr.* 36 *min.* 23.6 *sec.* P.M. *18.* 53 *min.* 48 *sec.* A.M., Jan. 2. *19.* 366.855+ *mi.*, Long. 76° 40′ 3″.3 W. *20.* 167° 48′ 45″ E. *21.* At Philadelphia, 1 *hr.* 23.8 *sec.*

Page 211. *12.* $45. *14.* $412.50.

Pages 212, 213. *6.* $337.33+. *9.* $1.80+, $1.21+, $2.06, $1.39. *11.* $48, $31.50, $65, 46.87\frac{1}{2}$, $25. *12.* $41.06, $49.46+, $45.11+, $3.88+. *13.* $869.33+, $605.33+, $1753.33+. *17.* $317.33+, $201.44+, $224.25, $487.66+, $19.17−, $12.83+, $279.44+.

Pages 222-224. *1.* 191 *cm.* 2. 278^{m}9. *3.* 329^{m}52. *4.* 58 *cm.* *5.* 2^{m}49, 8^{m}51. *6.* 1536^{m}188. *7.* 146^{m}304, 138^{m}686. *8.* 72 *cm.*, 7 *dm.* 83. *9.* 206^{m}01, 1730^{m}632. *10.* 6 *cm.*, 21^{m}. *11.* 2^{m}1, 109^{m}081. *12.* 1233^{Km}, 13^{Hm}71+. *13.* 753^{Dm}. *14.* 39^{Kg}6−. *15.* 24^{Km}02. *16.* 9$\frac{1}{2}$ *hr.* *17.* 92617. *19.* 2700 *cl.*, 2^{Dl}7. *20.* 3^{Dl}954, 0^{Hl}3954, 39054, 395 *dl.* 4. *21.* 69700^{l}, 6970000 *cl.*, 697000 *dl.* *22.* ℈3.

23. ½ *c.* *24.* 16.3 *c.* *25.* 2.24 *c.* *26.* 13½ *c.* *27.* $5.21. *29.* 27300^{Kg}, 27300000^{g}. *30.* 12^{Kg}45. *31.* 20^{T}91—, 5^{T}25—. *32.* $8395. *33.* 26½ *c.* *34.* $4720. *35.* 27^{Kg}. *36.* 36^{Dg}. *37.* 3500^{g}. *38.* 1930^{g}. *39.* 173^{Kg}. *40.* 261^{Kg}. *41.* 8^{Kg}24. *42.* No; it is too light. *43.* 77 *cl.* *44.* 1'685. *45.* 45½ *cl.* *46.* 1.86—.

Pages 232-234. *3.* 86. *4.* 644. *5.* $78.18. *6.* 8.43¾. *7.* $2.61. *8.* .12 *oz.* *9.* $3.29. *10.* $58. *11.* $0.58. *12.* $180. *13.* $66. *14.* $18. *15.* $547.80. *16.* $9.37½. *17.* $13.0755. *18.* $494. *19.* $226.50. *20.* $1.80. *21.* $180. *22.* $0.37. *23.* $37. *24.* $3.024. *25.* $14.6187. *26.* $0.105. *27.* $21. *28.* $9.80. *29.* 8 *c.* *30.* $9. *31.* ⅛ *c.* *32.* 8⅓ *m.* *33.* $9.625. *34.* $12.50. *35.* $2.3993. *36.* 1.6687. *37.* 2.5308. *38.* 155. *39.* 306. *40.* $7.61¼. *41.* 6.4416. *42.* $\frac{51}{400}$, $\frac{7}{120}$. *43.* $\frac{14}{25}$, $\frac{9}{25}$, $\frac{2}{5}$. *44.* 2, $\frac{5}{12}$, $\frac{7}{8}$. *45.* 1, 2½, 4. *46.* $\frac{7}{300}$, $\frac{7}{150}$, $\frac{7}{125}$. *47.* $\frac{171}{400}$, $\frac{273}{600}$, $\frac{63}{100}$. *48.* $\frac{3}{25}$, $\frac{3}{100}$, $\frac{4}{25}$. *49.* $\frac{11}{120}$, $\frac{143}{400}$, $\frac{44}{175}$. *50.* $\frac{111}{444}$, $\frac{122}{1300}$, $3\frac{453}{700}$. *51.* 33⅓. *52.* 20. *53.* 16⅔. *54.* 15$\frac{15}{16}$. *55.* 37½. *56.* 66⅔. *57.* 150. *58.* 53⅓. *59.* 100. *60.* 1. *61.* 50. *62.* ½. *63.* 150. *64.* 187½. *65.* 150. *66.* 110. *67.* 300. *68.* 160. *69.* 32. *70.* 1, 10000. *71.* 3080. *72.* 2450. *73.* 833⅓. *74.* 21.45. *75.* 532.50. *76.* 343¾. *77.* 2100. *78.* 3300. *79.* ¾. *80.* 40. *81.* 1825. *82.* 1920. *83.* 355⅝. *84.* 1033⅓. *85.* ⅓. *86.* $3700. *87.* $37. *88.* $12.33⅓. *89.* $234.54. *90.* 62½. *91.* $300. *92.* $120. *93.* $58.60. *94.* $36. *95.* $38.50. *96.* $125.60. *97.* $34.80. *98.* $80. *99.* $381.50. *100.* $\frac{90}{151}$. *101.* $500. *102.* $220.75+. *103.* $917.43+, *104.* 4.38—. *105.* $50. *106.* $99.01—. *107.* $51.68—. *108.* $139.42+. *109.* ⅔. *110.* $1.88+. *111.* $520. *112.* $28. *113.* $320. *114.* 760.42+ *yd.* *115.* 634.78+ *lb.* *116.* 385.55+ *ft.* *117.* 47.67—. *118.* 115.17—. *119.* 1⅛. *120.* 14$\frac{74}{100}$. *121.* 364. *122.* 650. *123.* 480. *124.* 148.39—. *125.* $26.63. *126.* $181.82—. *127.* $309.28—. *128.* $\frac{480}{1331}$. *129.* 4$\frac{84}{113}$. *130.* Any sum.

Pages 235-239. *1.* 434, 266. *2.* $81.396. *3.* $825. *4.* $5785. *5.* Lost $112.50. *6.* $4.40, $4.50, $5, $5.20, $4.60, $4.80. *7.* .0648, .060375, .0606, .0615, .0603, .06105, .063, .0627. *8.* $3. *9.* 10%, 90%. *10.* 10%. *11.* 25%. *12.* 33⅓, 11⅓, 233⅓. *13.* 25, 75. *14.* 33⅓. *15.* 100, 20, 10, 150. *16.* 25, 12½, 6¼, 37½. *17.* 13$\frac{1}{33}$. *18.* 15$\frac{25}{287}$. *19.* 28¾. *20.* 38+. *21.* $5. *22.* $250. *23.* $3.04+. *24.* 25 *c.* *25.* 20¾. *26.* $5.76. *27.* $4.35. *28.* 4. *29.* $22.73. *30.* Gain 12%. *31.* $48. *32.* 19$\frac{1}{21}$. *33.* 10% loss, 20% loss, cost 62½ *c.* *34.* Infinity %. *35.* $8888.89—. *36.* 16$\frac{4}{11}$, 14$\frac{1}{16}$. *37.* 108⅓. *38.* 6¾ *c.* *39.* $228.26. *42.* 20. *43.* 25. *44.* 14¼ *c.* *45.* 23$\frac{61}{199}$.

Pages 240-242. *1.* $7.20, $64.80. *2.* 7. *3.* 7. *4.* $7500. *5.* $17632. *6.* $1.25. *7.* $594.22½. *8.* 38$\frac{1}{4}$, $249.42. *9.* 68, $64600. *10.* $1285. *11.* $8.50, $994.50. *12.* $300. *13.* $2823.53. *14.* $604.17. *15.* 4½. *16.* 6. *17.* 60. *18.* 50. *19.* $1484.38—. *20.* 8544.63+. *21.* $5467.74, $182.26. *22.* $5727. *23.* 44$\frac{11}{111}$%.

Pages 244, 245. *1.* $2475. *2.* 15½. *3.* 50½. *5.* $90, $29250. *6.* .9, $13.50. *7.* $3563. *9.* 6$\frac{28}{117}$+. *11.* Real Estate.

Pages 246-249. *1.* Premium $12½. *2.* $50. *3.* Total Premium $48, total payment $49.75. *4.* $120. *5.* $96.25. *6.* $19424. *7.* $\frac{1}{500}$. *8.* $366.67—. *9* $1825. *10.* $846. *11.* ¾. *12.* 1¼%. *13.* $4210.67—. *14.* $4343.62+. *15.* $201.75. *16.* $144. *18.* $803.21+. *19.* $733.33+, A $5454.55—, B $6818.18+, C $2727.27+. *20.* $1691. *21.* $13233.33+. *22.* $7692.31—.

Pages 249-252. *2.* 1½ mills—, $3. *3.* $81, $16.20, $9, $1.80, $60, $12. *4.* $31500. *5.* $25000. *6.* $5599.10. *8.* $32.38. *10.* $2811.06. *11.* $12,670,357.75, $54,301,533.21. *12.* $50,314.29. *13.* $162,764.83. *14.* 1.620466+ *c.* *16.* $109.06, $10.26, $345.76. *17.* $7713.83, $28.58, $5.69, $11570.74+. *18.* $7.37.

Pages 254, 255. *2.* $1698.41. *3.* $2296.80. *4.* $611.10. *5.* $248.06. *6.* $826.50. *7.* $626.40. *8.* $157.93. *9.* $1175.60. *10.* $654.80.

Pages 258-260.* Sometimes the interest is here given, and sometimes the amount. *6.* $8 55+. *8.* $51.90. *9.* $73.50. *10.* $840.98—. *11.* $463.04—. *12.* $13.22½. *13.* $13.28—. *14.* $8.15—. *15.* $384.09+. *16.* $110.14½. *17.* $4.59+. *18.* $19.91—. *19.* $450. *20.* $10.23. *21.* $9.22. *22.* $132.37. *23.* $759.30. *24.* $15.82. *25.* $45.34. *26.* $0.955. *27.* $0.088. *28.* $504. *29.* $4.72. *30.* $22.46. *31.* $80. *32.* $3.50. *33.* $36.51. *34.* $455.39. *35.* $8.05. *36.* $4.66. *37.* $0.606. *38.* $24.90. *39.* $71.48. *40.* $0.46. *41.* $11.25. *42.* $45. *43.* $9.41. *44.* $179.64. *45.* $113.05. *46.* $15.40. *47.* $412.78. *48.* $28.75. *49.* $30.85. *50.* $37.

Pages 261-264. *2.* $41.60. *3.* $0.72. *4.* $0.18. *5.* $4.29. *6.* 6.81. *7.* $118.40. *8.* $1.46, $21.97. *9.* $14.16, $45.04. *10.* $400, $705.54. *11.* $10.82. *12.* $45.59. *13.* $10.55. *16.* $148.18. *17.* $614.05. *18.* $39.83. *19.* $788.88. *20.* $427.50. *21.* $1680. *22.* $757.50. *23.* $148.37, $98.03, $107.25.

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* Answers to practical business problems are often given to the nearest cent, *without further* indication. So, likewise, in other practical examples, the sign + *is not always* inserted when there is a remainder.

Pages 266, 267. *3.* \$109.08+. *4.* \$29.73+, \$37.53. *5.* \$3.94. *6.* \$4.64. *7.* \$7.84. *8.* \$68.07. *9.* \$136.60. *10.* \$14.93. *11.* \$3.07. *12.* \$0.34.

Pages 271, 272. *4.* \$42.67. *5.* \$36.09. *6.* \$134.98, \$78.22, \$61.02. *7.* \$53.22. *8.* \$99.51. *9.* \$101.77. *11.* \$15.50. *12.* \$394.44. *13.* \$7.21. *14.* \$1.35. *15.* \$54.12. *16.* \$724.68.

Page 273. *2.* \$662.34. *3.* \$393.63. *4.* \$108.24. *5.* \$490. *6.* \$441.82. *7.* \$986.31. *8.* \$95.07. *9.* \$320.96. *10.* \$643.46.

Pages 275–277. *3.* \$166.51. *4.* \$119.77. *5.* \$43.12. *6.* \$172.46.

Page 278. *2.* \$137.22. *3.* \$104.59. *4.* \$113.43. *5.* Aug. 20, 1874, \$163.65.

Page 280. *2.* \$650.55. *3.* \$379.11, \$380.81.

Pages 283, 284. *3.* \$225. *4.* \$6. *5.* \$470, \$475. *6.* \$2.75. *7.* \$54. *8.* The first requires \$337.75, and the second \$475. *9.* \$3.37½. *10.* \$3.75. *11.* 10%. *12.* \$3290.62½. *13.* \$1778.65.

Pages 285–288. *3.* \$86.30. *4.* \$171.94. *5.* \$550.46. *6.* \$804.36. *7.* \$314.28. *8.* \$19.08. *9.* \$381.59. *10.* \$1423.45. *11.* \$2516.26. *12.* Allowing grace, \$504.96. *13.* With grace, and discounting for $\frac{41}{360}$ *yr.*, \$6224.48. *14.* \$3.02, \$1.58, \$6.37, \$2.26.

Pages 290, 291. *2.* \$50.37. *3.* \$1018.41. *4.* \$30.36. *5.* \$322.26, \$153.98, \$756.62. *6.* \$74.80. *7.* \$553.58. *8.* \$2495.42. *9.* Yes. \$10.34.

Pages 293, 294. Reckoning calendar months and odd days as 360ths of a year, and no grace. *4.* \$300.93. *5.* \$209.17+. *6.* \$41.19+ less. *7.* \$328.35. *8.* \$310.34. *9.* \$290. *10.* \$300, \$325, \$371.43. *11.* \$230.78, \$250, \$285.71, \$300. *12.* \$4038.46. *13.* \$78.15. *14.* \$690. *15.* \$818.57. *16.* \$283.45. *17.* \$616.12.

Pages 298–300. *1.* \$582.50. *2.* \$345. *4.* \$1033.65. *5.* \$566.81. *6.* \$113.62½. *7.* \$876.35. *8.* \$1023.75, \$1018.44. *9.* \$529.90. *10.* \$2561.25, \$2546.46. *11.* \$5760. *12.* \$1037.25, \$1027.25. *13.* \$553.12½, 6$\frac{4}{9}$%. *16.* 3.87+%. *17.* State 7s.

Pages 303–305. *2.* \$1501.25. *3.* \$1824.09. *4.* \$2497.50. *5.* \$2500. *6.* \$1498.13. *7.* \$3003.75. *10.* \$1.01137, \$758.53, \$86.47. *11.* \$0.9748, \$487.40, \$179.61. *12.* 101.29%, 100.687%, 100.476%, 101.613%. *13.* 98.093%, 97.752%, 97.723%, 98.376%.

Page 306. *2.* \$1584.37½, \$1589.25. *3.* 2579.375. *4.* \$239.375.

Pages 308, 309. *3.* 10½%. *9.* 4 *yr.* 3 *mo.* 6 *da.*, 5 *yr.* 1 *mo.* 18 *da.*, 8 *yr.* 6 *mo.* 12 *da.* *10.* $520.23, $1250, $681.82. *11.* 2 *yr.* 3 *mo.* 18 *da.* *12.* 6%. *14.* $352.58, $354.97, $357.38. *15.* $500.08. *16.* 18⅛ − %. *17.* $21428.57. *18.* $50000. *19.* $32000. *20.* $60000.

Pages 315–317. *3.* $224. *4.* 1152 *bu.* *5.* $.130. *8.* $315.91. *9.* $490.20+. *10.* $1606.43. *11.* $244.80. *12.* 22 *min.* 48⅜ *sec.* *13.* 498 *yd.* 9½ *in.* *14.* 23 1/16 *da.* *15.* 12 *da.* *16.* 1089.4+*ft.* *17.* 3.1 *mi.* nearly. *18.* 185,485 *mi. per sec.* *19.* 7¾ nearly. *20.* 17 *yr.* 34 *da.* 21 *hr.* 32 *min.* 33 *sec.* *21.* 150 *ft.*

Pages 317, 318. *2.* 15, 20; 10, 25; 5, 30. *3.* 1⅝, 2⅜, 3⅛. *4.* 1⅛, 1 7/16, 3⅛, ⅞. *5.* $714⅞, $1428⅝, $2857⅛. *6.* $26⅜, $23⅛, $33⅛, $16⅜. *7.* A $100, B $200, C $400, D $500, E $700. *8.* A $40, B $45. *9.* A $102, B $138, C $150. *10.* A $120, B $120, C $40. *11.* A $24, B $32, C $36, D $28. *13.* $405 5/7, $270 10/17, $324 8/17.

Page 320. *2.* 11. *3.* 13⅝. *4.* 896. *5.* 1205 1/16. *6.* $77.73 *7.* 3 1/16 *wk.* *8.* 1⅛ *mo.* *9.* 10½. *10.* 72. *11.* 576. *12.* 24.

Pages 331, 332. *1.* 273 *ft.* *2.* 12.649, 208.71. *3.* 80, 113.137. *4.* $3^{Dm}1622$, $6^{Dm}3245$, $7^{Dm}7459$, 1 *Hm.*, 5 *Hm.*, $8^{Hm}3666$. *5.* 407.92 *rd.* by 203.96 *rd.* *6.* 1.41 *ft.*, 1.732 *ft.*, 2 *ft.*, 3.162 *ft.*, 3 *ft.*

1. 6, 9, $\sqrt{55}$, $\sqrt{\frac{1}{3}}$. *2.* 2⅞, 18, 5⅛, 9, 2⅜, 2 4/25, 2 9/27.

1. 357 *ft.*, 7.87 *ft.* *2.* 12.9 *in.*, 59.9+ *in.* *3.* 73. *4.* 5.039. *5.* 9 × 27 × 54. *6.* 1.26− *ft.*, 1.442+ *ft.*, 1.58+ *ft.*, 2, 3, 3.42− *ft.*

Pages 333–347. **(440)** *1.* 3 *ft.* 7.26 *in.* *2.* 10 *ft.* 11.59 *in.* *3.* 18 *ft.* .32 *in.* *4.* 29 *ft.* 5.53 *in.* *5.* 124.9− *rd.* *6.* 27 *ft.* 5.94 *in.* *7.* 17 *ft.* 10.65 *in.*, 19 *ft.* 2.74 *in.*, 22 *ft.* 7.52 *in.*, 35 *ft.* 9.24 *in.* **(442)** *1.* 13 *ft.* 1.09 *in.* *2.* 10 *ft.* 7.32 *in.* *3.* 1037¼ *in.* **(444)** *1.* 4 *yd.* *2.* 40 *yd.* *3.* 320. *4.* 1215, 2430. *5.* 2880. *6.* $34.32. *7.* 20. **(446)** *1.* 3. *2.* 19 2/11. *3.* 104.57+. *4.* 1.8154+. *5.* 2.0003+. **(447)** *1.* 16 2/3. *2.* 24½. **(448)** *3.* 418⅜, $5.86. *4.* 1275. *5.* 2095 *sq. ft.*, $37.71. *6.* $47.95. *7.* $34.075. *8.* 70. **(449)** *1.* 4.276, 7.33 *ft.* *2.* No. *3.* 23.758. *4.* 50.93−. *5.* 7.13 *rd.* **(450)** *1.* 43⅛ *yd.*, 40⅝ *yd.* *2.* $25.51, $25.96. *3.* 20, $6.00. *4.* 26. **(452)** *1.* ⅓. *2.* 5/16. **(453)** *1.* 3⅜. *2.* 172⅛ *bu.*, 800 *bu.*, 51⅛ *bu.* *3.* The first. *4.* 190.43+ *lbs.* **(454)** *1.* 640, 5. *3.* 55½. *4.* 4⅛, 8. *6.* 14 3/16. *7.* 43¾, 32 1/8, 21⅞, 37⅛, 28⅛, 18¾, 31¼, 23 7/16, 15⅝, 50, 37½, 25. *8.* 5 *cd. ft.* 15 *cu. ft.*, or 1 *cu. ft.* less than ¾ *cd.* *9.* 2 *cu. ft.* over ⅛ *cd.* *10.* 1173⅛. *11.* $28.89. *12.* $10.01. *13.* $39.12. *14.* 38500. *15.* 13⅛ *min.* *16.* 28181. *17.* 31674. **(455)** *1.* 196, 256, 676. *2.* *800, 547 (the* fractions are usually "saved" in Scribner's tables),

972,817. *3.* 1152, 960, 640, 1280. **(456)** *2.* 75.9 *cu. ft.*, 104.4 *cu. ft.*, 228.7 *cu. ft.* **(457)** *1.* 95.5—, 47.0016. *2.* 4406.4. *3.* 1997.368 *lbs.* *4.* 4712.4. **(464)** *1.* 350 *sq. ft.* *2.* 2,000,000 *cu. ft.* *3.* 106.75+. **(465)** *1.* 196,862,256.7296 *sq. mi.*, 259,726,937,378.5856 *cu. mi.* *2.* 12.566+ *sq. ft.*, 4.188+ *cu. ft.* **(466)** *1.* $1\frac{1}{8}$ times as strong, 4 times, 4 times. *2.* The latter, $\frac{108}{125}$. **(467)** *1.* 1° *F.* = $\frac{5}{9}$° *C.* *2.* 20°, 29$\frac{4}{9}$°, 28$\frac{8}{9}$° below 0. *3.* 84$\frac{1}{9}$°, 86°, $\frac{2}{9}$° below 0, 5°.

Pages 348-351. *1.* $23\frac{1}{3}$. *2.* $\frac{33}{50}$. *3.* $1\frac{17}{25}$. *4.* $32\frac{71}{378}$. *5.* $13\frac{5}{14}$. *6.* $\frac{61}{144}$. *7.* $\frac{194}{475}$. *8.* $\frac{5}{22}$. *9.* $5\frac{67}{185}$. *10.* 1812.59. *11.* 400,000,000. *12.* $10259\frac{8}{18}$. *13.* $129545\frac{5}{11}$. *14.* .00666. *15.* 400,000,000. *16.* $95\frac{168}{195}$. *17.* $\frac{1}{3}$. *18.* $\frac{8840}{19987}$. *19.* $2\frac{79}{1275}$. *20.* 1000. *21.* $\frac{78}{46875}$. *22.* 13.499. *23.* 13333.37331$\frac{1}{3}$. *24.* 4.1306+. *25.* $\frac{5}{6}$. *26.* 3.9686+. *27.* 10.2774+. *28.* 205.5176—. *29.* 8 and 6. *30.* Yes. *31.* No; $\sqrt{16}+\sqrt{4}$ is 6. while $\sqrt{16+4}$ is $\sqrt{20}$, or 4.47+. *32.* 1. *33.* 4.3958—. *34.* $\frac{13}{55}$. *35.* 24. *36.* $2\frac{4}{7}$. *37.* $6\frac{2}{3}$. *38.* $61\frac{1}{6}$. *39.* $\frac{5}{1625}$. *40.* .00006$\frac{2}{3}$. *41.* .0098$\frac{106}{163}$. *42.* 3.6305+. *43.* 3.2404—. *44.* 2. *45.* Yes; each is 2. *46.* No; the first is 2, and the second $\sqrt{12}$ = 3.4+. *47.* 1. *48.* 0. *49.* 8. *50.* 64. *51.* $\frac{3}{50}$. *52.* 3.5178+. *53.* .8216—. *54.* .8126—. *55.* $1\frac{43}{78}$. *56.* 3.7417—. *57.* .5477+. *58.* 1. *59.* 2.0207+. *60.* .7107—. *61.* $\frac{13}{75}$. *62.* $116\frac{164}{171}$. *63.* $1452\frac{44}{803}$. *64.* 1.3. *65.* 25. *66.* $\frac{288}{803}$. *67.* $\frac{2}{27}$. *68.* $37\frac{1}{27}$. *69.* $1\frac{13}{75}$. *70.* $73\frac{91}{125}$. *71.* $151\frac{1}{4}$. *72.* 3741.3258+. *73.* 148.401475. *74.* $1\frac{1151}{3024}$. *75.* 10.0259+. *76.* $\frac{5000}{29195}$. *77.* $\frac{8000}{48807}$. *78.* $\frac{2700}{80837}$. *79.* 1.21253. *80.* $1\frac{787}{5000}$. *81.* $\frac{87597}{617600}$. *82.* $\frac{2028}{3750}$. *83.* 27. *84.* $2\frac{2}{29}$, $2\frac{13}{58}$, $3\frac{2}{29}$. *85.* $2\frac{38}{101}$, $2\frac{14}{101}$, $7\frac{49}{101}$. *86.* $3\frac{6}{73}$, $1\frac{26}{73}$, $\frac{45}{73}$, $3\frac{69}{73}$. *87.* 500, 500, 250. *88.* $873.563, $1091.954, $1019.157, $815.325. *92.* $\frac{5775}{10759}$. *93.* $\frac{4181113}{20000000}$. *94.* $1\frac{1437}{2500}$. *95.* $\frac{227}{820}$. *96.* $\frac{7874}{12875}$. *97.* $\frac{15748}{17685}$. *98.* The ratio of 1 to .5144. *99.* 183.245+. *100.* .1385+.

Pages 351-360. *1.* 20, 50, $11\frac{27}{43}$. *2.* $183\frac{1}{3}$, 110, $157\frac{1}{7}$. *3.* $795.83. *4.* $202.84, $353.24, $253.27. *5.* $397.19. *6.* 20. *7.* $7\frac{4}{17}$. *8.* $1615.41. *9.* 38. *10.* 16 *yr.* 8 *mo.*, 14 *yr.* 3 *mo.* 13 *da.*, 12 *yr.*, 6 *mo.*, 10 *yr.* *11.* 14 *yr.* 2 *mo.* 13 *da.*, 7 *yr.* 3 *mo.* 5 *da.*, 17 *yr.* 8 *mo.* 1 *da.* *12.* $5060.54, $5031.37. *13.* 6. *14.* $8\frac{1}{3}$, $6\frac{2}{3}$. *15.* $1260. *16.* 3. *17.* $18000. *18.* $54259.36, $94.95. *19.* $0.0042, $14000. *20.* $8\frac{3}{25}$. *21.* $33\frac{1}{3}$. *22.* $3200. *23.* $12\frac{1}{2}$. *24.* 26. *25.* 175. *26.* $1697.92. *27.* $8. *28.* $189.47+. *29.* $33\frac{1}{3}$. *30.* 50, 100, $16\frac{2}{3}$. *31.* $\frac{1}{3}$ *gal.*, 1 *gal.*, $1\frac{1}{2}$ *gal.* *32.* $1287.03. *33.* 54. *34.* 64 *in.* *35.* $2560. *36.* $183.05. *37.* $\frac{64}{64}$ of itself, $2\frac{2}{3}$ times itself, 7 times itself. *38.* .21 of itself, .331 of itself, 3 times itself, 7 times itself. *39.* 55.78— *rd.* *40.* 27 *ft.* 5.9+ *in.* *41.* $9\frac{1}{4}$ *M.*, within 30 shingles. *42.* The deck will be 12 *ft.* by 18 *ft.*, the side rafters 14 *ft.* 5 *in.*+, and the corner rafters 18 *ft.* 9.1 *in.*+. *43.* $14\frac{30}{175}$. *44.* 2700 *lb.*

45. 648 *sq. in.* *46.* 1.154+ *in.* on the edge. *47.* 67½ *lb.* *48.* 4¼+. *49.* 10 acres. *50.* 5.642 *ft.* *51.* 6.59+ *ft.* *52.* 1,249,330—. *53.* 27 : 1331. *54.* 1.633—, 1.154+, 1.414+, 1.732+, 1. *55.* 3.634+, 3.175—, 2.52— *in.*, ⅛. *56.* 432 *lb.* *57.* 10, 15, 20 *ft.* *58.* 76 *yd.* *59.* $76.05. *60.* 1041. *61.* 41760. *62.* $60.25. *63.* 2.944 *qt.*, 6½ *qt.* *64.* 19.9. *65.* 15. *66.* 374⅝. *67.* $\frac{171}{1939}$, or 8.8%+. *68.* 181.6 *bu.*, 64^{Hl}, 4953.3—^{Kg}. *69.* 4 *o'clock* 17 *min.* 8 *sec.*+. *70.* 13° 4½′. *71.* 159⅜ *mi.* *72.* $108^{Kg}2$. *73.* 1.018. *74.* 1.842. *75.* 188^{Kg}. *76.* .8. *77.* 136.955 *lb.* *78.* 137.09 *lb.* *80.* 2,982,287,838,372,389,193,194. *81.* 48 *min.* past 4 P. M. *82.* 13½ *da.* *83.* ⅛ as long. *84.* 20 *da.* *85.* 15 *da.* *86.* A, 12 *da.*; B, 15; C, 60. *87.* 56 *oz.* *88.* He gained $263.31; the avoirdupois ounce being lighter than the Troy, he gained by selling by the former. *89.* 5^g16—, 6^g45. The silver franc really weighs 5^g. *90.* 26⅔°, 36⅔°, —33⅓°. *91.* 134°.6, .4° below 0. *92.* Wednesday, Monday. *95.* 1$\frac{19}{35}$. *96.* The German paid 5$\frac{15}{29}$%, and the French 4$\frac{76}{181}$%; the French in the ratio 2.05+ to 1.85+. *97.* $\frac{85}{21755}$ *in.* *98.* 16$\frac{4}{11}$%. *99.* The amounts are $700, $730, and $732.05. *100.* $51437.50.

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